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A generalization of Rejection Lemma of Drozd-Kirichenko

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1. Introduction.

Let R be a complete discrete valuation ring, which is fixed once for all as our base ring. Let K denote the quotient field of R. As for basic terminology such as R-lattice, R-order etc., we mostly follow that of [**CR**]. Let Λ be an R-order in a K-algebra $\tilde{\Lambda} := K\Lambda \simeq K \otimes_R \Lambda$, and let $\operatorname{Ind} \Lambda$ denote the set of isomorphism classes of indecomposable left Λ-lattices. For an overorder Γ of Λ in $\tilde{\Lambda}$, we may naturally consider $\operatorname{Ind} \Gamma$ as a subset of $\operatorname{Ind} \Lambda$. A subset $\mathscr{S}$ of $\operatorname{Ind} \Lambda$ will be called a *rejectable* subset if there is an overorder Γ such that $\mathscr{S} = \operatorname{Ind} \Lambda - \operatorname{Ind} \Gamma$. The map $\Gamma \mapsto (\operatorname{Ind} \Lambda - \operatorname{Ind} \Gamma)$ defines a bijection from the set of all overorders of Λ onto the set of all rejectable subsets of $\operatorname{Ind} \Lambda$. Indeed, the inverse map is given by $\mathscr{S} \mapsto \Lambda(\mathscr{S})$. Here, for any subset $\mathscr{S}$ of $\operatorname{Ind} \Lambda$, $\Lambda(\mathscr{S})$ is defined as the intersection $\bigcap_{L \in \operatorname{Ind} \Lambda - \mathscr{S}} O_l(L)$ of the left multiplier $O_l(L) := \{x \in \tilde{\Lambda} \mid xL \subseteq L\}$.

For any subset $\mathscr{S}$, $\Lambda(\mathscr{S})$ is an R-subalgebra of $\tilde{\Lambda}$ containing Λ, but is not necessarily an R-order of $\tilde{\Lambda}$. A subset $\mathscr{S}$ will be called *cofaithful* if $\Lambda(\mathscr{S})$ is an R-order of $\tilde{\Lambda}$. In particular, a rejectable subset $\mathscr{S}$ is always cofaithful. A subset $\mathscr{S}$ will be called *trivial* if $\Lambda(\mathscr{S}) = \Lambda$. While a subset $\mathscr{S}$ will be called *bounded* if the rational length $l(L) := \operatorname{length}_{\tilde{\Lambda}}(K \otimes_R L)$ is bounded on $\mathscr{S}$. When $\mathscr{S}$ is a singleton set, there is known a criterion (= determinable necessary and sufficient condition) for $\mathscr{S}$ to be rejectable, i.e. Rejection Lemma of Drozd-Kirichenko [**DK**].

1.1 D-K Rejection Lemma *$\mathscr{S} = \{P\}$ is rejectable if and only if P is bijective (= projective and injective) and $P \not\simeq \operatorname{rad} P$.*

1.1.1 Utility of D-K Rejection Lemma was well exhibited in [**DK**] where it was applied for Bass orders, and more generally quasi-Bass orders in semi-simple K-algebras. Further, in [**HN-1**], it was applied for Bass orders in non-semi-simple K-algebras.

1.1.2 Hijikata [**H**] studied also almost Bass orders, which is defined as a Gorenstein order Λ such that $O_l(\operatorname{rad} \Lambda)$ is also Gorenstein. He has shown that very precise results for almost Bass orders (including classification) can be derived by D-K Rejection Lemma, and suggested a possibility to extend Rejection Lemma for $\mathscr{S}$ with more than two points. Notably, the result of [**HN-2**] shows that, excepting for a small number of them (counted in representation type), each local order Λ of finite representation type has a minimal rejectable subset $\mathscr{S}$ consisting of four points of a definite shape, whose $\Lambda(\mathscr{S})$ is the unique minimal local overorder of Λ, and this is the reason, in a sense, why the Λ can be of finite representation type.