J. Math. Soc. Japan
Vol. 50, No. 3, 1988

# On the $C^\infty$-Goursat problem for some second order equations with variable coefficients

By Yukiko HASEGAWA

(Received Dec. 6, 1995)
(Revised Sept. 20, 1996)

## § 1. Introduction.

In this paper we study $C^\infty$-Goursat problem for the following $L$:

$$L = \partial_t \partial_x + a(t,x)\partial_y^2 + b(t,x)\partial_y + c(t,x), \tag{1.1}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, $\partial_y = \partial/\partial y$, for

$$(t,x,y) \in [0,\infty) \times \boldsymbol{R}^2 \quad \text{or} \quad (t,x,y) \in (-\infty,0] \times \boldsymbol{R}^2.$$

The coefficients $a(t,x)$, $b(t,x)$ and $c(t,x)$ are real valued $C^\infty$-functions, which are independent of $y$. For given $C^\infty$-functions $f(t,x,y)$, $g(x,y)$ and $h(t,y)$ with the compatibility condition $g(0,y) = h(0,y)$, the Goursat problem is to find a function $u(t,x,y)$ which satisfies

$$\text{(P)} \qquad \begin{cases} Lu = f(t,x,y) \in C^\infty_{(t,x,y)}, \\ u(0,x,y) = g(x,y) \in C^\infty_{(x,y)}, \\ u(t,0,y) = h(t,y) \in C^\infty_{(t,y)}, \quad \text{for } t \geqq 0 \text{ or } t \leqq 0. \end{cases}$$

We say that the Goursat problem (P) is $\mathscr{E}$-wellposed for $t \geqq 0$ (resp. for $t \leqq 0$) if for any data $\{f,g,h\} \in \mathscr{E}_{(t,x,y)} \times \mathscr{E}_{(x,y)} \times \mathscr{E}_{(t,y)}$ there exists a unique solution $u(t,x,y)$ of (P) belonging to $\mathscr{E}_{(t,x,y)}$ with $t \geqq 0$ (resp. $t \leqq 0$). In this case we also say that the Goursat problem for $L$ is $\mathscr{E}$-wellposed for $t \geqq 0$ (resp. for $t \leqq 0$). If (P) is $\mathscr{E}$-wellposed for $t \geqq 0$ (resp. for $t \leqq 0$) then it follows from Banach's closed graph theorem that the linear mapping $\{f,g,h\} \to u(t,x,y)$ is continuous from $\mathscr{E}_{(t,x,y)} \times \mathscr{E}_{(x,y)} \times \mathscr{E}_{(t,y)}$ to $\mathscr{E}_{(t,x,y)}$ for $t \geqq 0$ (resp. for $t \leqq 0$).

The $C^\infty$-Goursat problem with constant coefficients has been treated by several authors, for instance [**4**], [**5**], [**6**], and [**8**]. When the coefficients $a$, $b$ and $c$ are constant, we know that the necessary and sufficient condition for (P) to be $\mathscr{E}$-wellposed for both $t \geqq 0$ and $t \leqq 0$, is $a = b = 0$. In the case of variable coefficients what is the necessary condition for (P) to be $\mathscr{E}$-wellposed? It is the main problem that we study in this paper. On the other hand Nishitani [**9**] and Mandai [**7**] had also studied $C^\infty$-Goursat problem for general operators with variable coefficients. However our operator is excluded from their concern.