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On a class of multilinear oscillatory singular integral operators

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1. Introduction.

We will work on $\boldsymbol{R}^n$ $(n \geq 1)$. Let $\Phi(x) \in C^\infty(\boldsymbol{R}^n \setminus \{0\})$ be a real-valued function which satisfies

$$(1) \qquad |D^\alpha \Phi(x)| \leq B_1 |x|^{a-|\alpha|}, \quad |\alpha| \leq 3,$$

and

$$(2) \qquad \sum_{|\alpha|=2} |D^\alpha \Phi(x)| \geq B_2 |x|^{a-2},$$

where a is a fixed real number, B_1 and B_2 are positive constants. Let K_0 be a standard Calderón-Zygmund kernel. Define the oscillatory singular integral operator T by

$$(3) \qquad Tf(x) = \int_{\boldsymbol{R}^n} e^{i\Phi(x-y)} K_0(x-y) f(y)\, dy.$$

For the special case $\Phi(x) = |x|^a$, such operators have been studied by many authors (see [**1**], [**2**], [**7**], [**10**], for example). Recently, Fan and Pan [**6**] considered the operators defined by (3) with smooth phase functions satisfying (1) and (2). They showed that

THEOREM A. *Let* $1 < p < \infty$, T *be defined as in* (3). *Suppose that* Φ *satisfies* (1) *and* (2) *for some* $a \neq 0$. *Then* T *is bounded on* $L^p(\boldsymbol{R}^n)$ *with bound* $C(n,p)$.

THEOREM B. *Let* T *be defined as in* (3). *Suppose that* Φ *satisfies* (1) *and* (2) *for some* $a \neq 0, 1$. *Then* T *is a bounded operator on the Hardy space* $H^1(\boldsymbol{R}^n)$.

The purpose of this paper is to consider a class of multilinear operators related to the operators defined by (3). Let m be a positive integer, K be C^1 away from the origin and satisfy

$$(4) \qquad |K(x)| \leq C|x|^{-n}, \quad |\nabla K(x)| \leq C|x|^{-n-1},$$

and

$$(5) \qquad \int_{a<|x|<b} K(x) x^\alpha\, dx = 0, \quad \text{for any } 0 < a < |x| < b < \infty \text{ and } |\alpha| = m.$$

Let A have derivatives of order m in $\mathrm{BMO}(\boldsymbol{R}^n)$, $R_{m+1}(A; x, y)$ denote the $(m+1)$-th order

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