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On pluricanonical maps for threefolds of general type

By Meng Chen

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§ 1. Introduction

Let X be a nonsingular projective threefold of general type over the complex number field $\boldsymbol{C}$. It remains open whether there exists an absolute number $m(3)$ such that $\Phi_{|mK_X|}$ is a birational map onto its image when $m \geq m(3)$ for any X. Restricting interest to objects of nonsingular minimal threefolds of general type, Benveniste [**1**] got $m(3) = 9$ and then Matsuki [**9**] obtained $m(3) = 7$. In this paper, we want to show $m(3) = 6$.

Main Theorem. *Let X be a nonsingular projective threefold with nef and big canonical divisor K_X, then the* 6*-canonical map $\Phi_{|6K_X|}$ is a birational map onto its image.*

Throughout this paper, most our notations and terminologies are standard except the following which we are in favour of:
$:=$—definition;
$\sim_{lin}$—linear equivalence;
$\sim_{num}$—numerical equivalence.

§ 2. Proof of the Main Theorem

2.1 Kawamata-Viehweg's vanishing theorem. We will use the vanishing theorem in the following form.

Proposition 2.1 (Theorem 1.2 of [**5**]). *Let X be a nonsingular complete variety, $D \in \mathrm{Div}(X) \otimes \boldsymbol{Q}$. Assume the following two conditions:*
(1) *D is nef and big;*
(2) *the fractional part of D has the support with only normal crossings.*
Then $H^i(X, \mathcal{O}_X(\lceil D \rceil + K_X)) = 0$ for $i > 0$, where $\lceil D \rceil$ is the minimum integral divisor with $\lceil D \rceil - D \geq 0$.

2.2 Basic formula. Let X be a nonsingular projective threefold. For a divisor $D \in \mathrm{Div}(X)$, we have

$$\chi(\mathcal{O}_X(D)) = D^3/6 - K_X \cdot D^2/4 + D \cdot (K_X^2 + c_2)/12 + \chi(\mathcal{O}_X)$$

by Riemann-Roch theorem. The calculation shows that

$$\chi(\mathcal{O}_X(D)) + \chi(\mathcal{O}_X(-D)) = -K_X \cdot D^2/2 + 2\chi(\mathcal{O}_X) \in \boldsymbol{Z},$$

therefore $K_X \cdot D^2$ is an even integer, especially K_X^3 is even.

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