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Grassmann geometries on compact symmetric spaces of general type

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Introduction.

This study is a continuation of my papers [12], [13] and [14]. Let M be a compact simply connected riemannian symmetric space of dimension m (≥ 2), and s an integer such that $1 \leq s \leq m$. Let $G^s(T_pM)$ be the set of s-dimensional linear subspaces in a tangent space T_pM and denote by $G^s(TM)$ the Grassmann bundle over M with fibres $G^s(T_pM)$. For an arbitrary subset $\mathscr{V}$ in $G^s(TM)$ an s-dimensional connected submanifold S of M is called a *$\mathscr{V}$-submanifold* if at each point p of S the tangent space T_pS belongs to $\mathscr{V}$. The collection of $\mathscr{V}$-submanifolds, denoted by $\mathscr{S}(M, \mathscr{V})$, constitutes a *$\mathscr{V}$-geometry*. The term "Grassmann geometries" in the title is a collected name for such $\mathscr{V}$-geometries and it has been introduced in R. Harvey-H. B. Lawson [4].

We now consider the following $\mathscr{V}$-geometries. Let G be the isometry group of M. Then it acts transitively on M and at the same time acts on $G^s(TM)$ via the differentials of isometries. If as a subset $\mathscr{V}$ we take a G-orbit on $G^s(TM)$ by this action, the $\mathscr{V}$-geometry gives a class of submanifolds in M with congruent tangent spaces.

We moreover consider G-orbits of the following type. An s-dimensional linear subspace V in T_pM is called *strongly curvature-invariant* if it satisfies that

$$R_p(V, V)V \subset V \quad \text{and} \quad R_p(V^{\perp}, V^{\perp})V^{\perp} \subset V^{\perp},$$

where R denotes the curvature tensor on M and $V^{\perp}$ denotes the orthogonal complement of V in T_pM. We consider a G-orbit $\mathscr{V}$ through such a subspace V. The $\mathscr{V}$-geometry is also said to be of *strongly curvature-invariant type*. By a result on symmetric space a $\mathscr{V}$-geometry of strongly curvature-invariant type has a unique compact totally geodesic $\mathscr{V}$-submanifold, except the difference by isometries.

In the previous papers we have treated the cases that G is a simple Lie group, and have decided the $\mathscr{V}$-geometries which admit non-totally geodesic $\mathscr{V}$-submanifolds. In the present paper we treat a general case that the Lie group G is not necessarily simple, and we obtain a decomposition theorem for $\mathscr{V}$-submanifolds, thus, for $\mathscr{V}$-geometries of strongly curvature-invariant type. The theorem will clarify the structure of $\mathscr{V}$-geometries of strongly curvature-invariant type, and as a result it will give the classification of symmetric submanifolds in a general compact simply connected riemannian symmetric space.

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