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Initial-final value problems for ordinary differential equations and applications to equivariant harmonic maps

By Takeyuki NAGASAWA[1] and Keisuke UENO

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1. Introduction.

It is a fundamental problem to show the existence or nonexistence of a harmonic map between complete Riemannian manifolds. In the case of compact manifolds, a remarkable existence result is due to Eells-Sampson [**3**]. They showed that there exists a harmonic map if the target manifold has nonpositive sectional curvature. However, there is no general theory in the case the target manifold has positive sectional curvature. As for spheres, Smith [**5**] reduced the harmonic map equation to an ordinary differential equation and solving it, he constructed harmonic maps between spheres (see [**2**, **12**, **13**] for details and related topics).

This reduction technique works well even if manifolds are noncompact complete ones. Indeed, Urakawa and the second author [**11**] proved the existence of harmonic maps between noncompact cohomogeneity-one Riemannian manifolds. Here, a Riemannian manifold M is called a cohomogeneity-one Riemannian manifold if there exists a compact Lie group action G on M such that the quotient space M/G is a one-dimensional manifold. If a harmonic map between cohomogeneity-one Riemannian manifolds is invariant under these Lie group actions, then the harmonic map equation is reduced to an ordinary differential equation. Using this reduction, they showed the existence of harmonic maps between the complex hyperbolic spaces, the real hyperbolic spaces and the standard Euclidean spaces.

Our purpose of this paper is to study the ordinary differential equation (1.4)–(1.5) which appears in [**11**]. This equation has been already studied in [**10**], and proved the existence of a global and unbounded solution. In this paper, under different and more relaxed conditions, we present another proof of it and consider the finite time blow-up problem, which asserts the nonexistence of a harmonic map.

Let $f_i(t)$ and $h_i(r)$ $(i = 1, 2)$ be given functions defined on $[0, \infty)$ satisfying the following conditions.

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