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Equivariant algebraic vector bundles over cones with smooth one dimensional quotient

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Introduction.

This paper is concerned with aspects of the general problem of constructing and distinguishing equivariant algebraic vector bundles over a base space which is an affine variety with an algebraic action of a complex reductive group G i.e. an affine G-variety.

In previous papers (See references.) we have been interested in general aspects of equivariant stably trivial vector bundles over a base which is an arbitrary affine G-variety. In those papers special emphasis was placed on the case where the base is a representation. In this paper we introduce a new class of equivariant varieties as base space. For these we give a complete description of a naturally defined subset of stably trivial equivariant bundles and give applications.

There are two important classes of base spaces for equivariant vector bundles. These are homogeneous spaces and representations. In the case of homogeneous spaces all equivariant vector bundles are known and easy to describe. For representations, far less is known but an interesting picture is developing. What we do in this paper is to introduce a class of affine G-varieties called *weighted G-cones* which from the point of view of the G-action are somewhat more complex than homogenous spaces but far simpler than representations. We are able to describe some of the equivariant vector bundles over weighted G-cones in Theorem A. As an application we apply the results to describe families of equivariant vector bundles over representations. See Theorems B and C.

Here is some of the history which inspired this work. It illustrates the state of the subject. The history begins with two important problems—the Equivariant Serre Problem and the Linearity Problem.

LINEARITY PROBLEM. *Is every algebraic action of G on C^n conjugate to a linear action?*

Kambayashi in [**Ka**] conjectured an affirmative answer and treated this in a paper with Russell in [**KR**]. This stimulated a lot of research. Somewhat later Bass-Haboush [**BH1**] tied this in with the Equivariant Serre Problem and showed how to produce a negative answer to the Linearity Problem from a negative answer to the Equivariant Serre Problem.

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