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Spaces of discrete shape and c-refinable maps that induce shape equivalences

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Introduction.

In [15], following a Cantor completion process, the authors give a complete, non-Archimedean metric (or ultrametric) on the set of shape morphisms between two unpointed compacta (compact metric spaces) X, Y, written $Sh(X, Y)$. The ultrametric spaces so constructed allow to rediscover some of the more important invariants in shape theory and to introduce many others. It is clear that the construction given in [15] can be translated to the pointed case, consequently, as a particular case, we obtain a complete ultrametric that induces a norm on the shape groups of a compactum Y.

Let (X, x_0) and (Y, y_0) be pointed compacta. We will assume Y to be embedded in the Hilbert cube Q. Let $i_\varepsilon : Y \to B(Y, \varepsilon)$ be the inclusion. For any pair $f, g : (X, x_0) \to (Q, y_0)$ of maps, take $F(f, g) = \inf\{\varepsilon > 0 : f \cong g \text{ in } B(Y, \varepsilon) = Y_\varepsilon\}$ ($\cong$ means the pointed homotopy relation).

It is clear that (pointed) approximative maps (see [3]) $\{f_k\} : (X, x_0) \to (Y, y_0)$ correspond with F-Cauchy sequences and that (pointed) homotopic approximative maps are equivalent F-Cauchy sequences.

Given $\alpha, \beta \in Sh((X, x_0), (Y, y_0))$ and F-Cauchy sequences $\{f_k\}$, $\{g_k\}$ in the classes of α, β respectively, the formula $d(\alpha, \beta) = \lim_{k\to\infty} F(f_k, g_k)$ produces a well defined complete, non-Archimedean metric in $Sh((X, x_0), (Y, y_0))$ such that the composition of pointed shape morphisms induces uniformly continuous maps between the spaces involved. This fact provides many new pointed shape invariants (see [15] for details in the unpointed case).

PROPOSITION 1 ([15]). *Given $\alpha, \beta \in Sh((X, x_0), (Y, y_0))$, $d(\alpha, \beta) < \varepsilon$ if and only if $S(i_\varepsilon) \circ \alpha = S(i_\varepsilon) \circ \beta$, as pointed morphisms ($S$ denotes the shape functor).*

In order to simplify notation *we suppress base points consistently* until section 2.

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