J. Math. Soc. Japan
Vol. 49, No. 3, 1997

Modified Nash triviality theorem for a family of zero-sets of weighted homogeneous polynomial mappings

Dedicated to Professor Yasutoshi Nomura on his 60th birthday

By Satoshi KOIKE

(Received July 13, 1994)
(Revised July 28, 1995)

§ 0. Introduction.

Let $\alpha=(\alpha_1, \cdots, \alpha_n)$ be an n-tuple of positive integers. Assume that the greatest common divisor of the integers α_j is 1. Let $\boldsymbol{N}$ denote the set of positive integers, and let $f:(\boldsymbol{R}^n, 0)\rightarrow(\boldsymbol{R}, 0)$ be a polynomial function defined by

$$f(x)=\sum_{\beta}A_{\beta}x_1^{\beta_1}\cdots x_n^{\beta_n}\ (A_{\beta}\neq 0,\ \beta_1, \cdots, \beta_n\in\boldsymbol{N}\cup\{0\}).$$

We say that f is *weighted homogeneous of type* $(\alpha_1, \cdots, \alpha_n; L)$ $(\alpha_1, \cdots, \alpha_n, L\in\boldsymbol{N})$, if

$$\alpha_1\beta_1+\cdots+\alpha_n\beta_n=L \qquad \text{for any} \quad \beta=(\beta_1, \cdots, \beta_n).$$

Let J be an open interval, and $t_0\in J$. Let $f_t:(\boldsymbol{R}^n, 0)\rightarrow(\boldsymbol{R}^p, 0)$ be a polynomial mapping where each $f_{t,i}$ is weighted homogeneous of type $(\alpha_1, \cdots, \alpha_n; L_i)$ $(1\leqq i\leqq p)$ for $t\in J$. We define a mapping $F:(\boldsymbol{R}^n\times J, \{0\}\times J)\rightarrow(\boldsymbol{R}^p, 0)$ by $F(x, t)=f_t(x)$. Assume that F is a polynomial mapping (or of class C^2). It is well-known that the following fact holds under these assumptions:

FACT. If $f_t^{-1}(0)\cap\Sigma f_t=\{0\}$ for any $t\in J$ (where Σf_t denotes the singular points set of f_t), then $(\boldsymbol{R}^n\times J, F^{-1}(0))$ is topologically trivial i.e. there exists a t-level preserving homeomorphism $\sigma:(\boldsymbol{R}^n\times J, \{0\}\times J)\rightarrow(\boldsymbol{R}^n\times J, \{0\}\times J)$ such that

$$\sigma((\boldsymbol{R}^n\times J, F^{-1}(0)))=(\boldsymbol{R}^n\times J, f_{t_0}^{-1}(0)\times J).$$

REMARK 1. Results generalizing this fact have been obtained in [**2**], [**5**]. But it seems that the fact itself was recognized by many mathematicians a good while ago.

This research was partially supported by Grant-in-Aid for Scientific Research (No. 06640139), Ministry of Education, Science and Culture, Japan.