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Totally geodesic boundaries are dense in the moduli space

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Let F be a closed, oriented surface such that the genus of each component of F is greater than 1. In this paper, we will study the subset $\mathcal{R}(F)$ of the moduli space $\mathcal{M}(F)$ such that a hyperbolic structure $s\in\mathcal{M}(F)$ is an element of $\mathcal{R}(F)$ if there exists a compact, connected, oriented hyperbolic 3-manifold M with totally geodesic boundary and admitting an orientation-preserving isometry $\varphi:\partial M\rightarrow F(s)$, where ∂M is assumed to have the orientation induced naturally from that on M. Note that $\mathcal{R}(F)$ is a countable subset of $\mathcal{M}(F)$.

First, consider the special case where F consists of two components each of which is homeomorphic to a given closed surface Σ of genus >1. In Fujii [**3**], it is implicitly seen that, for any $s\in\mathcal{M}(\Sigma)$, one can construct a compact, connected, oriented, hyperbolic 3-manifold M with totally geodesic, two-component boundary such that one component is arbitrarily close to $\Sigma(s)$ in $\mathcal{M}(\Sigma)$ and the other is to $\Sigma(\bar{s})$ (see Lemma 1 in §2 for the explicit proof based on the circle-packing argument in Brooks [**2**]). Here, $\bar{s}\in\mathcal{M}(\Sigma)$ denotes the hyperbolic structure on Σ admitting an orientation-reversing isometry $\varphi:\Sigma(s)\rightarrow\Sigma(\bar{s})$. This implies that the closure of $\mathcal{R}(F)$ in $\mathcal{M}(F)$ contains the skew diagonal $\Delta_{\mathrm{skew}}(\Sigma)=\{(s,\bar{s})\,;\,s\in\mathcal{M}(\Sigma)\}$ of $\mathcal{M}(F)=\mathcal{M}(\Sigma)\times\mathcal{M}(\Sigma)$.

In this paper, we will consider a more general case and prove the following theorem.

THEOREM. *Suppose that* $F=\Sigma_{1}\sqcup\cdots\sqcup\Sigma_{t}$ *is any closed, oriented surface such that the genus of each component* Σ_{i} *is greater than* 1. *Then,* $\mathcal{R}(F)$ *is dense in* $\mathcal{M}(F)=\mathcal{M}(\Sigma_{1})\times\cdots\times\mathcal{M}(\Sigma_{t})$.

McMullen's results in [**9**], [**10**] play important roles in our proof of Theorem. Especially, the argument in [**10**] for skinning maps is well applicable to construct a compact, connected, hyperbolic 3-manifold M by joining "long" hyperbolic 3-manifolds associated to any $s_{i}\in\mathcal{M}(\Sigma_{i})$ $(i=1,\cdots,t)$ so that ∂M is totally geodesic and arbitrarily close to $\Sigma_{1}(s_{1})\sqcup\cdots\sqcup\Sigma_{t}(s_{t})$ in $\mathcal{M}(F)$.

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