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On the decomposition of lattices over orders

By Hiroaki HIJIKATA

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0. Introduction.

We shall extend two basic theorems on decomposition of lattices over orders—'Roiter-Jacobinski Divisibility Theorem' and 'Jacobinski-Swan Cancellation Theorem'—to an arbitary R-order Λ over an arbitary Dedekind domain R. The point is that we do not assume the ambient algebra $A=K\Lambda$ to be separable over the quotient field K of R.

0.0. As for terminology, we mostly follow that of [1] and [2]. However, for a maximal ideal P of R, the suffix P like R_P always denotes the P-adic completion rather than the localization.

A left Λ-lattice L' will be called a local direct summand of another Λ-lattice L if L'_P is a direct summand of L_P for any maximal ideal P.

Write $KL \gg KL'$ if every A-indecomposable direct summand of KL' occurs strictly oftener in KL than in KL'.

Write $M \sim L$ if $L_P \cong M_P$ for any P.

THEOREM 1 (Roiter-Jacobinski type Divisibility). *Suppose that L' is a local direct summand of L. Then*

(i) *L has a direct summand M' such that $M' \sim L'$.*

(ii) *If $KL \gg KL'$, then L' itself is a direct summand of L.*

THEOREM 2 (Jacobinski-Swan type Cancellation). *Assume that the K-algebra $B=\mathrm{End}_A KL$ has the "strong approximation". Then the following cancellation law* (c) *holds.*

(c) *If L' is a local direct summand of $nL=L\oplus L\oplus \cdots \oplus L$ (n-times), then $L\oplus L' \cong M\oplus L'$ implies $L\cong M$.*

0.1. Remark on Theorem 1. (i) is known if A is separable over K (cf. [1] 31.12.) (ii) is known if A is separable over K and moreover K is a global field, i.e., K is a finite extension of the rational number field $\boldsymbol{Q}$ or of the rational function field $\boldsymbol{F}_q(T)$ (cf. [1] 31.32, [4], [6].)

The current proof of (i) heavily depends on the existence of maximal orders, while the proof of (ii) depends on Jordan-Zassenhaus Theorem.