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A lower bound for sectional genus of quasi-polarized manifolds

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Introduction.

Let X be a smooth projective variety over $\boldsymbol{C}$ with $\dim X=n$, and L an ample (resp. a nef and big) Cartier divisor. Then (X, L) is called a polarized (resp. a quasi-polarized) manifold.

For this (X, L), the sectional genus of L is defined to be a non negative integer valued function by the following formula ([**Fj2**]):

$$g(L)=1+\frac{1}{2}(K_X+(n-1)L)L^{n-1},$$

where K_X is the canonical divisor of X.

Then there is the following conjecture:

CONJECTURE 1 (p. 111 in [**Fj3**]). *Let (X, L) be a quasi-polarized manifold. Then $g(L)\geqq q(X)$, where $q(X)=h^1(X, \mathcal{O}_X)$ (called the irregularity of X).*

In [**Fk1**], we treat $\dim X=2$ case. But if $\dim X\geqq 3$, the problem seems difficult. So we consider the following conjecture:

CONJECTURE 2. *Let (X, L) be a quasi-polarized manifold, Y a normal projective variety with $1\leqq\dim Y<\dim X$, and $f: X\rightarrow Y$ a surjective morphism with connected fibers. Then $g(L)\geqq h^1(\mathcal{O}_{Y'})$, where Y' is a resolution of Y.*

Of course Conjecture 2 follows from Conjecture 1. The hypothesis of Conjecture 2 is natural because X has a fibration in many cases (Albanese fibration, Iitaka fibration, etc.).

In this paper, we consider Conjecture 2. In particular, we study $\dim Y=1$ or some special cases of $\dim Y\geqq 2$. Using some results with respect to Conjecture 2, we study Conjecture 1.

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