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On polynomials which determine holomorphic mappings

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§ 1. Introduction.

We say that two meromorphic functions f and g on $\boldsymbol{C}$ share the value a if the zeros of $f-a$ and $g-a$ ($1/f$ and $1/g$ if $a=\infty$) are the same. In [**N**], R. Nevanlinna showed the following two results:

THEOREM A. *If two distinct nonconstant meromorphic functions on* $\boldsymbol{C}$ *share four values by counting multiplicities, then one is a Möbius transformation of the other.*

THEOREM B. *If two nonconstant meromorphic functions on* $\boldsymbol{C}$ *share five values, then they are identical.*

These results are interesting from the viewpoint of determining meromorphic or holomorphic functions but it is troublesome to check four or five pairs of values of meromorphic or holomorphic functions. Also there are results in [**F1**] and [**F2**] which show the uniqueness of holomorphic mappings into complex projective spaces.

Recently, H.-X. Yi proved the following:

THEOREM C. *Let* n *and* m *be two positive integers such that* n *and* m *have no common factor and* $n>2m+4$. *Let* a *and* b *be two nonzero constants such that the algebraic equation* $P(w)=w^{n}+aw^{n-m}+b=0$ *has no multiple roots. If two nonconstant entire functions* f *and* g *satisfy* $P(g)=\alpha P(f)$ *for some entire function* α *without zeros, then* $f=g$.

In this, it is enough for determining holomorphic functions to check only one pair of holomorphic functions. So, the author asks the following two questions:

QUESTION 1. *Do there exist polynomials* P_{n} *of variables* $z_{1}, \cdots, z_{n}$ *with the property*:

(P) *if two algebraically nondegenerate holomorphic mappings* f *and* g *of* $\boldsymbol{C}$ *into* $\boldsymbol{C}^{n}$ *satisfy* $P_{n}(g)=\alpha P_{n}(f)$ *for some entire function* α *without zeros, then* $f=g$.

QUESTION 2. *Do there exist homogeneous polynomials* H_{n} *of variables* $w_{0}, \cdots, w_{n}$ *with the following property*: