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Transverse structure of Lie foliations

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0. Introduction.

This paper deals with the problem of the realization of a given Lie algebra as transverse algebra to a Lie foliation on a compact manifold.

Lie foliations have been studied by several authors (cf. [**4**], [**5**], [**6**], [**12**], [**17**]). The importance of this study was increased by the fact that they arise naturally in Molino's classification of Riemannian foliations (cf. [**14**]).

To each Lie foliation are associated two Lie algebras, the Lie algebra $\mathcal{G}$ of the Lie group on which the foliation is modeled and the structural Lie algebra $\mathcal{H}$. The latter algebra is the Lie algebra of the Lie foliation $\mathcal{F}$ restricted to the closure of any one of its leaves. In particular, it is a subalgebra of $\mathcal{G}$. We remark that $\mathcal{H}$ is canonically associated to $\mathcal{F}$, but $\mathcal{G}$ is not.

Thus two interesting problems are naturally posed: the *realization problem* and the *change problem*.

The *realization problem* is to know which pair of Lie algebras ($\mathcal{G}$, $\mathcal{H}$), with $\mathcal{H}$ a subalgebra of $\mathcal{G}$, can arise as transverse and structural Lie algebras, respectively, of a Lie foliation $\mathcal{F}$ on a compact manifold M.

This problem is closely related to the following Haefliger's problem (see [**9**]): given a subgroup Γ of a Lie group G, is there a Lie G-foliation on a compact manifold M with holonomy group Γ?

The present formulation of the *realization problem* in terms of Lie algebras was first considered in [**10**], and [**7**] made a very detailed study of Lie flows of codimension 3. But a complete classification was not obtained because of the following open questions:

i) Let $\mathcal{G}_7^k$ be the family of Lie algebras for which there is a basis $\{e_1, e_2, e_3\}$ such that

$$[e_1, e_2]=0, \quad [e_1, e_3]=e_1, \quad [e_2, e_3]=ke_2, \quad k\in[-1, 0)\cup(0, 1].$$

For which k is there a Lie $\mathcal{G}_7^k$-flow on a compact manifold with basic

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