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L^2 harmonic forms and stability of minimal hypersurfaces

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0. Introduction.

The Bernstein conjecture states that any complete minimal graph in E^{m+1} is a hyperplane. This was proved to be true for $m\leqq 7$ by Bernstein [**2**] ($m=2$), De Giorgi [**5**] ($m=3$), Almgren [**1**] ($m=4$) and Simons [**12**] ($m\leqq 7$); and false for $m\geqq 8$ by Bombieri, De Giorgi and Giusti [**3**].

On the other hand, do Carmo and Peng [**6**] and Fischer-Colbrie and Schoen [**8**] showed that complete orientable and stable minimal surfaces in E^3 are planes. Palmer [**10**] studied a topological restriction of a complete minimal hypersurface M in E^{m+1} which implies instability of M. This topological restriction is related to the existence of nonzero L^2 harmonic 1-forms by Dodziuk's result [**7**].

We denote the space of all L^2 harmonic p-forms on a complete orientable Riemannian manifold M by $\mathcal{H}^p(M)$. $\mathcal{H}^p(M)$ consists of p-forms which are closed and coclosed by a theorem of Andreotti and Vesentini. It is well known that $\mathcal{H}^p(E^m)=\{0\}$ for all p; $0\leqq p\leqq m$.

A complete minimal graph in E^{m+1} is minimizing, and any minimizing minimal hypersurface is stable. Therefore, concerning the Bernstein problem we can pose the following problem: *For a complete orientable and stable minimal hypersurface M in E^{m+1}, does $\mathcal{H}^p(M)=\{0\}$ hold for all p; $0\leqq p\leqq m$.*

The case where $m=2$ is trivial by the result of do Carmo and Peng, Fischer-Colbrie and Schoen as stated in the above. A catenoid shows that the assumption of stability in our problem is essential. Here we have the following:

THEOREM A. *Let $M\subset E^{m+1}$ be a complete orientable and stable minimal hypersurface. If $m\leqq 4$, then $\mathcal{H}^p(M)=\{0\}$ holds for all p; $0\leqq p\leqq m$.*

Palmer [**10**] used the norm of an L^2 harmonic 1-form on M to define a variation vector field and proved that a complete orientable minimal hypersurface $M\subset E^{m+1}$ admitting a nontrivial L^2 harmonic 1-form is unstable. So, Theorem A for the case where $m=3$ is due to Palmer [**10**]. To prove Theorem A it suffices to show the following: Let $M\subset E^5$ be a complete, orientable minimal hypersurface;