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Uniqueness of the solution of non-linear singular partial differential equations

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Introduction.

The existence and the uniqueness of the solution of non-linear singular partial differential equations of the form

$$\text{(E)} \qquad \left(t\frac{\partial}{\partial t}\right)^m u = F\left(t,\ x,\ \left\{\left(t\frac{\partial}{\partial t}\right)^j\left(\frac{\partial}{\partial x}\right)^\alpha u\right\}_{\substack{j+|\alpha|\leq m \\ j<m}}\right)$$

were discussed in Gérard-Tahara [1], [2]; though, the uniqueness in [2] can be applied only to the solution with

$$\text{(0.1)} \qquad \left(t\frac{\partial}{\partial t}\right)^j u(t,\ x) = O(t^s) \quad (\text{as } t\to 0 \text{ uniformly in } x) \qquad \text{for } j=0,\ 1,\ \cdots,\ m-1$$

for some $s>0$.

In this paper, the author will prove the uniqueness of the solution of (E) under the following weaker assumption:

$$\text{(0.2)} \qquad \left(t\frac{\partial}{\partial t}\right)^j u(t,\ x) = O\left(\frac{1}{(-\log t)^s}\right) \quad (\text{as } t\to 0 \text{ uniformly in } x) \qquad \text{for } j=0,\ 1,\ \cdots,\ m-1$$

for some $s>0$.

The motivation for such an improvement will be illustrated in the following example.

EXAMPLE. Let us consider

$$\text{(0.3)} \qquad t\frac{\partial u}{\partial t} = \lambda u + u\frac{\partial u}{\partial x},$$

where $(t,\ x)\in \boldsymbol{C}\times\boldsymbol{C}$ and $\lambda\in\boldsymbol{C}$. Then:

(1) $u\equiv 0$ is a solution of (0.3).

(2) By the method of the separation of variables we can see that (0.3) has solutions of the form