J. Math. Soc. Japan
Vol. 48, No. 4, 1996

Some functional equations and Picard constants of algebroid surfaces

Dedicated to Professor Mitsuru Nakai on his 60th birthday

By Kiyoshi NIINO and Kazuya TOHGE

(Received May 25, 1994)
(Revised Oct. 14, 1994)

1. Introduction.

When we intend to calculate the Picard constants of algebroid surfaces and study analytic mappings among algebroid surfaces with large Picard constants, we have to consider some functional equations. For example, the first author [**7**], Ozawa-Sawada [**11**], [**12**], [**13**] and Sawada-Tohge [**14**], etc. (cf. [**10**]) considered special cases of the following functional equation:

$$\sum_{\mu=0}^{m} a_\mu(z)e^{\mu H(z)} = f(z)\sum_{\nu=0}^{n} b_\nu(z)e^{\nu L(z)}.$$

The purpose of this paper is to study the above equation and to give an application.

First we shall prove

THEOREM 1. *Let H and L be non-constant entire functions with $H(0)=L(0)=0$, $a_m=b_n=1$, a_μ $(\mu=0, 1, \cdots, m-1)$ and b_ν $(\nu=0, 1, \cdots, n-1)$ meromorphic functions with $a_0\not\equiv 0$, $b_0\not\equiv 0$ and f a meromorphic function. Further suppose that*

$$T(r, a_\mu) = S(r, e^H) \quad \mu = 0, \cdots, m-1, \tag{1.1}$$

$$T(r, b_\nu) = S(r, e^L) \quad \nu = 0, \cdots, n-1, \tag{1.2}$$

and

$$N(r, 0, f)+N(r, \infty, f) = o(m(r, e^H)+m(r, e^L)) \quad r\to\infty \tag{1.3}$$

outside a set of finite measure. If $m\geqq n\geqq 1$, $d=(m, n)$, $m=pd$, $n=qd$ and the identity

$$\sum_{\mu=0}^{m} a_\mu(z)e^{\mu H(z)} = f(z)\sum_{\nu=0}^{n} b_\nu(z)e^{\nu L(z)} \tag{1.4}$$

holds, then we have one of the following two cases: