Buekenhout geometries of rank 3
which involve the Petersen graph

By Thomas MEIXNER

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1. Introduction.

Consider the diagram $(c^*P)$: $o--o$. $P$.

Here, the symbol $o--o$ stands for the circle geometry with 4 points and
$P$--$P$ for the geometry of the Petersen graph. We will determine all simply
connected geometries $G$ with this diagram and flag-transitive automorphism
group. It will turn out that there are exactly five simply connected ones. One
of them is related to the alternating group of degree 6, two of them are related
to the Mathieu groups $M_{11}$ and $M_{13}$, all these three are finite.

One is related to the symmetric group of degree 9 and to the sporadic
group He, and one to the group $SO_{5}(5)$, and we do not know, whether they are
finite or infinite.

To be precise, we will prove the following theorem.

**THEOREM.** Let $\mathcal{G}$ be a connected, simply connected geometry with diagram
$(c^*P)$ and flag-transitive automorphism group $G$. Then $\mathcal{G}$ is one of the geomet-
tries $G_6$, $G_8$, $G_{10}$, $G_{11}$ or $G_{13}$ defined in the next section, and $G$ is isomorphic to one
of the groups $G_{10}$, $G_{16}$ (if $\mathcal{G}$ is $G_6$), or $G_{11}$ (if $\mathcal{G}$ is $G_{10}$), or $G_{12a}$, $G_{12b}$, $G_{13c}$, $G_{13d}$,$G_{13e}$ (if $\mathcal{G}$ is $G_{11}$), or $G_{9a}$, $G_{9b}$ (if $\mathcal{G}$ is $G_8$), or $G_{5a}$, $G_{5b}$ (if $\mathcal{G}$ is $G_{10}$), all defined in
the last section, respectively.

Here, $G_{11}$ is a geometry with 66 points and automorphism group $G_{11}=M_{11}$, $G_{12}$ is a geometry with 4752 points and automorphism group $G_{12d}$ (resp. $G_{12e})=(A_4\times M_{12})2$ and projects onto a geometry for $M_{12}$, $G_6$ is a geometry with 6480 points and automorphism group $G_{45}=3(A_5\times A_4)2$, while we do not know, whether the geometries $G_6$ and $G_8$ are finite or infinite. They project onto finite geomet-
tries for $SO_{5}(5)$ and $\Sigma_9$ respectively and have automorphism groups $G_{9a}$ and $G_{9b}$ respectively.

The remark on the automorphism groups of the examples is almost trivial: the pairs $(\mathcal{G}, \text{Aut}(\mathcal{G}))$ have to appear in the list, hence one has only to check in
every case, which of the groups acting on the same geometry $\mathcal{G}$ is the "biggest
one".