J. Math. Soc. Japan
Vol. 48, No. 3, 1996

Gradient estimates for a quasilinear parabolic equation of the mean curvature type

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(Received Sept. 7, 1994)

1. Introduction.

In this paper we are concerned with the gradient estimates of solutions to the initial boundary value problem of the quasilinear parabolic equation

$$u_t - div\{\sigma(|\nabla u|^2)\nabla u\} = 0 \quad \text{in } \Omega\times[0, \infty), \tag{1.1}$$

$$u(x, 0) = u_0(x) \text{ and } u(x, t)|_{\partial\Omega} = 0 \quad \text{for } t \geqq 0, \tag{1.2}$$

where Ω is a bounded domain in R^N with a smooth, say C^3 class, boundary $\partial\Omega$ and $\sigma(v)$ is a function like $\sigma(v)=1/\sqrt{1+v}$.

When $\sigma(v)=|v|^{(p-2)/2}$, $p\geqq 2$, Alikakos and Rostamian [**1**] derived an estimate for $\|\nabla u(t)\|_\infty$ for the solutions of the equation with Neumann boundary condition, which includes a smoothing effect and decay properties. The argument can be applied to the case of Dirichlet problem. In [**1**], a strong coerciveness condition on $-div\{\sigma(|\nabla u|^2)\nabla u\}$ is used essentially and the mean curvature type nonlinearity $\sigma(v)=1/\sqrt{1+v}$ is excluded.

Recently, Engler, Kawohl and Luckhaus [**2**] have treated the problem (1.1)-(1.2) for a class of $\sigma(v)$ including $\sigma(v)=|v|^{(p-2)/2}$ and $1/\sqrt{1+v}$ and derived estimates for $\|\nabla u(t)\|_q$, in particular if $\sigma'(v)\geqq\varepsilon_0>0$, the decay estimate

$$\|\nabla u(t)\|_q \leqq \|\nabla u_0\|_q e^{-\lambda t}, \quad \lambda>0, \tag{1.3}$$

for any $q\geqq 2$. In [**2**], however, no result concerning smoothing effect nor decay estimate for $\|\nabla u(t)\|_\infty$ is given.

The object of this paper is to derive an estimate for $\|\nabla u(t)\|_\infty$ to the problem (1.1)-(1.2) with $\sigma(v)$ like $1/\sqrt{1+v}$. Our result includes both of smoothing effect and exponential decay. More precisely, we prove

$$\|\nabla u(t)\|_\infty \leqq C\|\nabla u_0\|_{p_0} t^{-\mu} e^{-\lambda t} \tag{1.4}$$

for $p_0>3N/2$ ($p_0\geqq 3$ if $N=1$), where λ is a positive constant and $\mu=N/(2p_0-3N)$.

As in [**1**] and [**2**] (see Serrin [**9**]) we make a certain geometric condition on $\partial\Omega$, which is essential for our argument. Such a condition is useful even for some type of quasilinear wave equations ([**6**]).