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A remark on the exotic free actions in dimension 4

By Masaaki UE

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Donaldson's polynomial invariants ([**2**]) are powerful tools for studying smooth 4-manifolds. For example the diffeomorphism types of elliptic surfaces with positive geometric genus were completely classified by them ([**4**], [**20**], [**21**]). The examples of smooth closed 1-connected noncomplex 4-manifolds with infinitely many smooth structures were first given by [**9**] and then were constructed by various methods ([**6**], [**13**], [**16**], [**26**]). In this paper we will give some examples of infinitely many exotic 4-manifolds whose universal coverings are mutually diffeomorphic. In fact we will show that the fundamental group of any spherical 3-manifold other than the 3-sphere acts freely on certain 4-manifolds in infinitely many different ways so that their orbit spaces are exotic (Main Theorem). Throughout this paper we denote the $K3$ surface by K, and for any finite group G we denote by $|G|$ the order of G. For any closed oriented 4-manifold X, we denote by $b_2^+=b_2^+(X)$ the rank of the maximal positive subspace $H^+=H^+(X)$ of the intersection form q_X of X, and nX denotes the connected sum of n copies of X.

MAIN THEOREM. *Let G be the fundamental group of any spherical 3-manifold other than the 3-sphere. Let $X=(2n-1)\boldsymbol{CP}^2\#(10n-1)\overline{\boldsymbol{CP}}^2$ or $X=nK\#(n-1)S^2\times S^2$. Then if n is divided by $|G|$ and $|G|<n$, there exist infinitely many smooth orientation-preserving free G-actions on X such that their orbit spaces are mutually homeomorphic but non-diffeomorphic to each other.*

In §1 we will construct the manifolds which will be the orbit spaces for the actions in Main Theorem. These manifolds are connected sums of rational homology 4-spheres and 1-connected 4-manifolds with $b_2^+>1$ which are derived from certain elliptic surfaces. In §2 and §4 we will describe the simple invariants for these manifolds to distinguish their diffeomorphism types. In §3 the list of $SO(3)$-representations for the above G will be given for the estimates of the simple invariants in §4. In §5 we will complete the proof of Main Theorem. Here we note that the argument in §4 is similar to that in [**15**] in which the connected sums of $\boldsymbol{Z}_2$ homology 4-spheres and some manifolds with