J. Math. Soc. Japan
Vol. 48, No. 2, 1996

The group ring of $GL_n(q)$ and the q-Schur algebra

By Mitsuhiro TAKEUCHI*

(Received Dec. 22, 1992)
(Revised Jun. 6, 1994)

Introduction.

Dipper-James [**5**] have introduced the q-Schur algebra $S_q(n)$ to study representations of $GL_n(q)$ in non-describing characteristic. The q-Schur algebra is a q-analogue of the usual Schur algebra, and its representations are equivalent to polynomial representations of quantum general linear group [**3**]. Dipper-James [**5**] have established an interesting relationship between representations of $GL_n(q)$ and the q-Schur algebra $S_q(n)$. They deal with not only unipotent representations but also cuspidal representations. In this paper, we restrict to unipotent representations and show there is a shorter realization of the Dipper-James correspondence in this case.

Let KG be the group algebra of $G=GL_n(q)$ over the field K whose characteristic does not divide q. Let B be the upper-triangular matrices and let $M=KG[B]$ the left ideal generated by $[B]$, the sum of all elements in B. Let I_M be the annihilator of M in KG. By *unipotent representations* of G, we mean left KG/I_M modules. Let **mod** KG/I_M be the category of all left KG/I_M modules.

Let λ be a partition of n. James [**9**] defines the Specht module S_λ and its irreducible quotient D_λ. Both are left KG/I_M modules, and the set of D_λ for all partitions λ of n exhausts all irreducible unipotent representations of G. On the other hand, Dipper-James [**6**] define the q-Weyl module W_λ and its irreducible quotient F_λ, which are left $S_q(n)$ modules. The purpose of this paper is to prove:

THEOREM. *Assume K has a primitive p-th root of* 1. *There is an idempotent E in KG/I_M satisfying the following properties*:

(a) *The algebra $E(KG/I_M)E$ is isomorphic to the q-Schur algebra $S_q(n)$.*

(b) *The functor $V\mapsto EV$ gives a category equivalence from* **mod** KG/I_M *to* **mod** $S_q(n)$.

(c) *Let λ be a partition of n, and let λ' be its dual partition. Under the category equivalence of* (b), *the KG/I_M module S_λ (resp. D_λ) corresponds to the $S_q(n)$ module $W_{\lambda'}$ (resp. $F_{\lambda'}$).*

* Supported in part by the Sumitomo Foundation grant 93-103-624.