J. Math. Soc. Japan
Vol. 48, No. 2, 1996

Extreme points and linear isometries of the domain of a closed *-derivation in $C(K)$

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(Received Sept. 24, 1993)
(Revised May 11, 1994)

§1. Introduction.

Unbounded derivations in non-commutative C^*-algebras have been studied in detail by many authors, which are closely related to mathematical physics and especially are one of the natural frameworks for quantum dynamics in operator algebra context ([**3, 4, 5, 14, 17, 18, 21, 22**]).

In commutative C^*-algebras, unbounded derivations, which were studied systematically by Sakai in [**21**], are also very important and interesting object to study, because it plays a role of certain differential structure of underlying space ([**1, 2, 9, 11, 12, 24**]). Indeed, known examples are given by (partial) differentiation on spaces with some differential structure.

Since the differentiation d/dt on the space $C^{(1)}([0, 1])$ of continuously differentiable functions on $[0, 1]$ is a typical example of closed derivations, for any closed derivation δ in a commutative unital C^*-algebra $C(K)$ (K: a compact Hausdorff space) we may regard the domain $\mathfrak{D}(\delta)$ of δ as a generalization of the Banach space $C^{(1)}([0, 1])$. Moreover, if $\mathfrak{D}(\delta)=C(K)$, δ is bounded and hence $\delta\equiv 0$. Thus, we wish to look for unified approach to deal with $C(K)$, $C^{(1)}([0, 1])$ and several other spaces of differentiable functions together.

Properties of the domains of closed derivations (for example, functional calculus) have been investigated by several authors. $\mathfrak{D}(\delta)$ becomes Banach algebras under several graph norms. Moreover, it was shown that $\mathfrak{D}(\delta)$ with a closed *-derivation δ is a Šilov algebra by Sakai ([**22**]), and other interesting properties of $\mathfrak{D}(\delta)$ as a Banach algebra have been studied by Batty, Goodman and Tomiyama ([**1, 2, 9, 24**]). We are also interested in some interplays between properties of $\mathfrak{D}(\delta)$ as a Banach space (or a Banach algebra) and the structure of δ.

On the other hand, a well-known Banach-Stone theorem [**8**] states that surjective linear isometries of $C(K)$ are induced by homeomorphisms of K and this theorem was extended to more general case by Novinger, Okayasu and Takagaki ([**15, 16**]). Moreover, the structure of surjective linear isometries of