J. Math. Soc. Japan
Vol. 48, No. 1, 1996

On the capacity of singularity sets admitting no exceptionally ramified meromorphic functions

By Kikuji MATSUMOTO and Toshiko KUROKAWA

(Received April 19, 1994)

1. Introduction.

For a totally disconnected compact set E in the extended z-plane $\hat{C}$, we denote by M_E the totality of meromorphic functions each of which is defined in the domain complementary to E and has E as the set of transcendental singularities. A meromorphic function $f(z)$ of M_E is said to be exceptionally ramified at a singularity $\zeta \in E$, if there exist values w_i, $1 \leqq i \leqq q$, and positive integers $\nu_i \geqq 2$, $1 \leqq i \leqq q$, with

$$\sum_{i=1}^{q}\left(1-\frac{1}{\nu_i}\right) > 2,$$

such that, in some neighborhood of ζ, the multiplicity of any w_i-point of $f(z)$ is not less than ν_i. Recently, we have shown that, for Cantor sets E with successive ratios $\{\xi_n\}$ satisfying $\xi_{n+1}=o(\xi_n^2)$, any function of M_E cannot be exceptionally ramified at any singularity $\zeta \in E$ (Theorem in [**5**]). The capacity (in this note, capacity means always logarithmic capacity) of these Cantor sets E is zero, because they satisfy the necessary and sufficient condition

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \log \frac{1}{\xi_n} = \infty$$

to be of capacity zero.

The purpose of this note is to give Cantor sets E of positive capacity improving the above theorem. We shall prove

THEOREM. *Let E be a Cantor set with successive ratios $\{\xi_n\}$ satisfying the condition*

$$\xi_{n+1} = o(\xi_n^{r_0}), \qquad r_0=(1+\sqrt{33})/4,$$

then any function of M_E cannot be exceptionally ramified at any singularity $\zeta \in E$.

We set $\xi_{n+1}=\xi_n^r$ $(n=1, 2, 3, \cdots)$ with r, $r_0<r<2$. Then $\{\xi_n\}$ satisfies the condition of the theorem and