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A theorem of Chevalley type for prehomogeneous vector spaces

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Introduction.

Let G be a complex reductive group, acting linearly on a vector space V. Assume that V is G-prehomogeneous, i.e., V has a dense G-orbit. Let $\boldsymbol{C}[V]_{G,\phi}=\{f\in\boldsymbol{C}[V]\mid f(gv)=\phi(g)f(v)\}$ and f be its non-zero element. Then it is known [1] that there exists a unique G-orbit O_1 which is closed in $\Omega:=f^{-1}(\boldsymbol{C}^{\times})$. Let T be a maximal torus of the isotropy subgroup $H:=G_{v_1}$ of G at $v_1\in O_1$, $N:=N_G(T)$ the normalizer of T in G, $G':=N/T$, and $V':=V_T=\{v\in V\mid tv=v$ for any $t\in T\}$. We can show that ϕ induces a character of G', which we shall denote by the same letter ϕ. Define $\boldsymbol{C}[V']_{G',\phi}$ in the same way as above.

The purpose of this note is to prove the following two theorems.

THEOREM A. (1) *V' is G'-prehomogeneous. More precisely, the G'-orbit of v_1 is open dense in V'.*

(2) *The isotropy subgroup G'_{v_1} of G' at v_1 is finite.*

(3) *The restriction $\boldsymbol{C}[V]\to\boldsymbol{C}[V']$ induces an isomorphism $\boldsymbol{C}[V]_{G,\phi}\tilde{\to}\boldsymbol{C}[V']_{G',\phi}$.*

THEOREM B. *Assume that H is finite. Take $h\in H$, and let $\langle h\rangle$ be the finite cyclic group generated by h. Put $N'':=N_G(\langle h\rangle)$, $G'':=N''/\langle h\rangle$, and $V'':=V_{\langle h\rangle}$. Then*

(1) *V'' is G''-prehomogeneous. More precisely, $G''\cdot v_1$ is open dense in V''.*

(2) *If $h\neq 1$, $|G''_{v_1}|<|G_{v_1}|$.*

(3) *Take a rational character ϕ of G and a non-zero relative invariant $f\in\boldsymbol{C}[V]_{G,\phi}$. Then ϕ induces a character of G'', which we shall denote by the same letter ϕ, and the restriction $\boldsymbol{C}[V]\to\boldsymbol{C}[V'']$ induces an isomorphism $\boldsymbol{C}[V]_{G,\phi}\tilde{\to}\boldsymbol{C}[V'']_{G'',\phi}$.*

NOTATION. If a group Γ acts on a set X, X_Γ denotes the set of Γ-fixed points, and Γ_x denotes the isotropy subgroup of Γ at $x\in X$. For two subsets $A, B\subset\Gamma$, $A^B:=\{b^{-1}ab\mid a\in A, b\in B\}$. We write a^B for $\{a\}^B$. The meaning of