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Expansion growth of smooth codimension-one foliations

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0. Introduction.

The entropy of foliations is defined by Ghys, Langevin and Walczak ([**G-L-W**]) as follows. Let $\mathcal{F}$ be a codimension q foliation of class C^0 on a compact manifold M. Fixing a finite foliation cover $\mathcal{U}$ of $(M, \mathcal{F})$, we obtain the holonomy pseudogroup $\mathcal{H}$ of local homeomorphisms of $\boldsymbol{R}^q$ induced by $\mathcal{U}$. We define an integer $s_n(\varepsilon)$ ($n \in \boldsymbol{N}$, $\varepsilon>0$) to be the maximum cardinality of (n, ε)-separating sets with respect to the holonomy pseudogroup $\mathcal{H}$. Then $s_n(\varepsilon)$ is monotone increasing on n and monotone decreasing on ε. The *entropy* $h(\mathcal{F}, \mathcal{U})$ of the foliation $\mathcal{F}$ is defined by the following formula:

$$h(\mathcal{F}, \mathcal{U}) = \lim_{\varepsilon \to 0} \limsup_{n \to \infty} \frac{1}{n} \log s_n(\varepsilon).$$

When we fix a sufficiently small positive real number ε, we notice that the monotone increasing map $s_n(\varepsilon)$ with respect to n represents the degree of the expansion of the foliation. In [**E1**], we considered the growth type of $s_n(\varepsilon)$ defined in the growth type set which is an extension of the usual growth type set (cf. [**H-H2**]) and we proved that the growth type of $s_n(\varepsilon)$ depends only on $(M, \mathcal{F})$. Therefore it becomes a topological invariant for foliations. We call it the *expansion growth* of $(M, \mathcal{F})$. By computing the expansion growth of several typical codimension 1 foliation of class C^0, we showed that the expansion growth of codimension 1 foliation of class C^0 takes uncountably many values.

In this paper, we compute the expansion growth of codimension 1 foliations of class C^2. The main result of this paper is the following.

THEOREM. *Let $\mathcal{F}$ be a transversely oriented codimension* 1 *foliation of class C^2 on a compact manifold M. Let K be an $\mathcal{F}$-saturated set.*

(1) *If $\bar{K}$ has a resilient leaf, then $\eta(K)=[e^n]$.*

(2) *If $\bar{K}$ has no resilient leaf and* $\mathrm{level}(K)<\infty$, *then $\eta(K)=[n^{\mathrm{level}(K)}]$.*

(3) *Otherwise, $\eta(K)=[1, n, n^2, \cdots]$.*

Here $\eta(K)$ means the expansion growth of $(M, \mathcal{F})$ on K and the notation $[\cdot]$ means the growth type defined in section 1 and $\mathrm{level}(K)$ means supremum