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The Dirac operator on space forms of positive curvature

By Christian BÄR

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1. Introduction.

Riemannian spin manifolds carry an important natural operator, for *Dirac operator*. The Dirac operator is an elliptic differential operator of first order acting on spinor fields, hence its spectrum is discrete point spectrum if the underlying manifold is compact. An excellent introduction to the general theory of Dirac operators can be found in [**15**]. The relation between the spectrum and the geometry of the manifold is currently an object of intense research. Explicit calculation of the spectrum is possible only for very nice manifolds. For example, for homogeneous spaces the calculation can be reduced to representation theoretic computations which still can be very hard, see [**2**]. To the author's knowledge the first explicit calculation was done by Friedrich in [**9**] for the flat torus to demonstrate the dependence of the Dirac spectrum on the choice of spin structure.

In this paper we study the Dirac spectrum of the sphere and of its quotients. Ikeda obtained analogous results for the Laplace operator on spherical space forms in a series of papers [**10**]-[**14**]. In [**10**] he calculates the spectrum of the Laplace operator acting on functions, in [**14**] he does the same for the Laplace operator acting on p-forms. In [**12**] and [**13**] he constructs non-isometric examples with the same Laplace spectrum.

We begin with the calculation of the Dirac spectrum on the standard sphere. Sulanke already did this in her unpublished thesis [**17**] using the representation theoretic methods mentioned above. But the necessary computations in her work are lengthy and it seemed desirable to find a simpler way to do it. Our main tool is the use of Killing spinors. Killing spinors are spinor fields satisfying a certain highly over-determined differential equation. Generically, they do not exist, but on the standard sphere they can be used to trivialize the spinor bundle. In this trivialization the calculation can be carried out without too much pain. The eigenvalues on S^n turn out to have a very simple form, they are given by $\pm(n/2+k)$, $k \geqq 0$ (Theorem 1).