J. Math. Soc. Japan
Vol. 48, No. 1, 1996

Periodic stability of solutions to some degenerate parabolic equations with dynamic boundary conditions

By Toyohiko AIKI

(Received Sept. 10, 1993)
(Revised Mar. 9, 1994)

0. Introduction.

This paper is concerned with a degenerate parabolic equation

$$u_t - \Delta\beta(u) = f \quad \text{in } Q := (t_0, \infty)\times\Omega \tag{0.1}$$

with dynamic boundary condition

$$\begin{cases} \dfrac{\partial\beta(u)}{\partial\nu} + \dfrac{\partial V}{\partial t} + h = 0 \\ V = \beta(u) \end{cases} \quad \text{on } \Sigma := (t_0, \infty)\times\Gamma, \tag{0.2}$$

where $t_0 \in \boldsymbol{R}$ or $t_0 = -\infty$; Ω is a bounded domain in $\boldsymbol{R}^N$ ($N \geqq 1$) with smooth boundary $\Gamma := \partial\Omega$; $(\partial/\partial\nu)$ denotes the outward normal derivative on Γ; $\beta : \boldsymbol{R} \to \boldsymbol{R}$ is a given nondecreasing function; f and h are given functions on Q and Σ, respectively. In this paper, we denote by "SP on (t_0, ∞)" the system $\{(0.1), (0.2)\}$.

Equation (0.1) represents the enthalpy formulation of the Stefan problem, when

$$\beta(r) = \begin{cases} c_1(r-1) & \text{for } r \geqq 1, \\ 0 & \text{for } 0 < r < 1, \\ c_2 r & \text{for } r \leqq 0 \end{cases}$$

for some positive constants c_1, c_2. For the physical interpretation of boundary condition (0.2) we quote Langer [**11**] and Aiki [**1**]. As far as initial-boundary value problems for (0.1) with usual boundary conditions are concerned, there are some interesting results (e.g., [**16**, **14**, **13**]) dealing with existence and uniqueness of solutions. Recently, problems with similar boundary conditions were discussed by Mikelič-Primicerio [**12**] and Primicerio-Rodrigues [**15**].

In Aiki [**1**], the existence and uniqueness of a weak solution of