J. Math. Soc. Japan
Vol. 47, No. 4, 1995

Self-similar diffusions on a class of infinitely ramified fractals

By Hirofumi OSADA

(Received Feb. 25, 1993)
(Revised Nov. 29, 1993)

§ 0. Introduction.

In this paper we construct self-similar diffusions on a class of infinitely ramified (self-similar) fractals.

Construction of self-similar diffusions on finitely ramified fractals has been done by Goldstein [**7**], Kusuoka [**12**], Barlow-Perkins [**6**] and Kumagai [**11**] for the Sierpinski gasket, and Lindstrøm [**14**] for the nested fractals.

As for infinitely ramified fractals, Barlow-Bass [**1**, **2**, **3**, **4**] and Barlow-Bass-Sherwood [**5**] studied the two dimensional Sierpinski carpet. Although some strong estimates of transition probability densities were obtained, the self-similarity and uniqueness of their Brownian motions were not known. Recently Kusuoka-Zhou [**13**] have constructed self-similar diffusions on the recurrent fractals; a class of fractals containing the two dimensional Sierpinski carpet.

One crucial step to study the infinitely ramified fractals in the above works was to use the quantities such as resistance and Poincaré constants. We however take a different approach: We first consider the equation (2.9) of hitting probabilities in the fasion of Lindstrøm [**14**];

$$\Phi(\mathbf{q}) = \mathbf{q} \qquad (\mathbf{q} \in \mathbf{Q}_{G,H}(\boldsymbol{F})). \tag{2.9}$$

Here $\mathbf{Q}_{G,H}(\boldsymbol{F})$ is the set consisting of hitting probabilities. Then we construct self-similar diffusions from its solutions.

In the case of nested fractals, $\mathbf{Q}_{G,H}(\boldsymbol{F})$ can be regarded as a compact convex set in $\boldsymbol{R}^n$ and the map Φ is continuous by the geometrical symmetry of fractals. Accordingly Lindstrøm could solve (2.9) by applying Brouwer's fixed point theorem. If the fractal is infinitely ramified, $\mathbf{Q}_{G,H}(\boldsymbol{F})$ becomes infinitely dimensional and it seems difficult at present to use fixed point theorems.

To solve the equation (2.9), we reduce the problem to the existence of an approximate solution $\mathbf{q}\in\mathbf{Q}_{G,H}(\boldsymbol{F};\, \mathcal{F})$ and obtain the solution $\mathbf{r}$ by taking $\mathbf{r}=\lim_{n\to\infty}\Phi^n(\mathbf{q})$ (Theorem 2.1). We also prove that such an approximate solution exists if the fractal has a nice surjection to another fractal where a self-similar