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On Varea's conjecture

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1. Main result.

Unless stated otherwise, Lie algebras mentioned in this paper are always assumed to be finite dimensional over algebraically closed field F.

For $x \in L$, denote $C(x)$ the centralizer of x in L. L is called to be centralizer nilpotent (abbreviated c. n.) provided that the centralizer $C(x)$ is nilpotent for all nonzero $x \in L$. Such algebras have been studied by Benkart and Isaacs in [**1**].

For $x \in L$, denote $E_L(x)$ the Engle subalgebra of L determined by x, i.e., the Fitting null-component of $\mathrm{ad}\, x$ in L. L is called to be Engel subalgebraically anisotropic (abbreviated E. a.) if every proper Engel subalgebra $E_L(x)$ of L has no any ad-nilpotent element of L. E. a. Lie algebras have been studied by Varea in [**2**].

In many special cases, [**2**] has proved that E. a. Lie algebras are c. n.. It is conjected that only E. a. simple Lie algebras are $sl(2, F)$, $W_p(F)$ and $sl(3, F)/F \cdot 1$, where $\mathrm{char}\, F = p > 0$. Under this conjecture, [**2**] has proved that every E. a. Lie algebra is c. n..

The aim of this short paper is to prove Varea's conjecture. In fact, the way used here is more direct than that of [**2**]. We will prove the following

THEOREM. *Let L be a Lie algebra over an algebraically closed field F. Then L is E. a. if and only if L is c.n..*

2. The proof of Theorem.

Sufficiency can be follow upon application of Theorem 2.5 of [**1**]. The main effort in the following will be to prove the necessity. Some preliminaries will be needed.

Let U be an abelian subalgebra of L. Then L can be decomposed into direct sum of weight spaces L_λ. That is, $L = \sum_\lambda L_\lambda$, where L_λ is the largest subspace of L on which $\mathrm{ad}\, u - \lambda(u)$ is nilpotent for all $u \in U$. Let $K_\lambda = \{\eta \in L \mid \mathrm{ad}\, u(\eta) = \lambda(u)\eta$, for all $u \in U\}$, $K = \sum_\lambda K_\lambda$. Then K is a subalgebra of L, for $[K_\lambda, K_\mu] < K_{\lambda+\mu}$.