J. Math. Soc. Japan
Vol. 47, No. 3, 1995

Strong multihomotopy and Steenrod loop spaces

By Antonio GIRALDO and Jose M. R. SANJURJO*)

(Received Oct. 21, 1993)

0. Introduction.

The strong shape category of metric compacta was introduced in 1973 by J. B. Quigley [**19**], although some notions related to strong shape were already considered by D. Christie [**6**] and T. Porter [**18**]. In particular, Christie defined the strong shape groups. In 1976 D. A. Edwards and H. M. Hastings [**11**], motivated by work of T. A. Chapman [**5**], obtained a category isomorphism between the strong shape category of compacta K in the pseudo-interior of the Hilbert cube, Q, and the proper homotopy category of their complements $Q-K$. Strong shape was extended to arbitrary topological spaces by F. W. Bauer [**1**] and Edwards and Hastings [**11**]. General information about the strong shape category of compacta is contained in the papers [**9**] by J. Dydak and J. Segal and [**3**] by F. W. Cathey. The first of them presents a geometric study of strong shape based on the notion of contractible telescope. The second one gives an account of several different approaches. We shall use in this paper the approach to strong shape given by J. B. Quigley [**19**] or, in a more general form, that given by Y. Kodama and J. Ono [**14**], [**15**] under the name of fine shape.

All the existing descriptions of the strong shape category of compacta use external elements to introduce the basic notion of strong shape. Compacta are generally assumed to lie in the Hilbert cube or in a convenient ambient space, like a manifold or a polyhedron, and maps take values in neighborhoods of the compacta in the ambient space. In other descriptions, compacta are presented as inverse limits of ANR systems and maps are defined between the systems and not directly between the compacta themselves.

We present in this paper a new description of strong shape. We eliminate all the external elements in our approach and obtain an intrinsic description of the strong shape category of compacta, completing in this way the program that was started in [**20**] and [**21**] for standard shape.

We use in our approach the theory of multivalued maps. Strong shape morphisms are characterized as homotopy classes of fine multivalued maps and

*) The second author is partially supported by CAICYT.