J. Math. Soc. Japan
Vol. 47, No. 2, 1995

L^p-mapping properties of functions of Schrödinger operators and their applications to scattering theory

By Arne JENSEN and Shu NAKAMURA

(Received Dec. 28, 1992)
(Revised July 19, 1993)

§ 1. Introduction.

In the first part of this paper, we study operators $f(H)$ and $e^{-itH}f(H)$ in $L^p(\boldsymbol{R}^d)$, where $H=-\Delta+V(x)$ is a Schrödinger operator defined primarily as a self-adjoint operator in $L^2(\boldsymbol{R}^d)$. For $H_0=-\Delta$, mapping properties of $f(H_0)$ between L^p-spaces and norm estimates for $e^{-itH_0}f(H_0)$ follow from the theory of Fourier multipliers. One of our goals is to extend these results to a fairly large class of Schrödinger operators $H=H_0+V(x)$. To attain this goal we use several tools, including properties of the Schrödinger semigroup: e^{-tH}, the spaces $l^p(L^q)$ which are sometimes called amalgams of l^p and L^q, commutator estimates, and a result (Theorem 2.4) which can be viewed as a version of the Beurling-Carlson theorem on Fourier multipliers (see [**BTW**]).

Throughout this paper we suppose the potential $V(x)$ satisfies the following condition:

ASSUMPTION (A). V is real-valued function on $\boldsymbol{R}^d$, and it is decomposed as $V(x)=V_+(x)-V_-(x)$ such that $V_\pm\geqq 0$, $V_+\in K_d^{\mathrm{loc}}$ and $V_-\in K_d$, where K_d is the Kato class of potentials.

For the sake of completeness, we recall the definitions of K_d and K_d^{loc} (cf. Simon [**S**: Section A2] for the detail):

DEFINITION. $V\in K_d$, if:

$$\text{For } d\geqq 3,\quad \lim_{r\to 0}\sup_{x\in\boldsymbol{R}^d}\int_{|x-y|\leq r}\frac{|V(y)|}{|x-y|^{d-2}}dy=0\,;$$

$$\text{For } d=2,\quad \lim_{r\to 0}\sup_{x\in\boldsymbol{R}^d}\int_{|x-y|\leq r}\log\{|x-y|^{-1}\}|V(y)|dy=0\,;$$

$$\text{For } d=1,\quad \sup_{x\in\boldsymbol{R}^d}\int_{|x-y|\leq 1}|V(y)|dy<\infty\,.$$