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Asymptotic expansions of the solutions to a class of quasilinear hyperbolic initial value problems

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0. Introduction.

Let us consider the initial value problem related to the following quasi-linear positive symmetric strictly hyperbolic system:

$$A_0(u)\frac{\partial}{\partial t}u+\sum_{\nu=1}^{n}A_\nu(u)\frac{\partial}{\partial x_\nu}u+B(u)u=0. \tag{0.1}$$

Thus, $A_0(u), \cdots, A_n(u)$ are $m\times m$ symmetric matrices depending smoothly on $u\in\boldsymbol{R}^m$, and $A_0(u)$ is positive definite while $B(u)$ may be any $m\times m$ smooth matrix. Strict hyperbolicity means that, for any $\xi=(\xi_1, \cdots, \xi_n)\neq 0$, the matrix

$$M(u, \xi)=\sum_{\nu=1}^{n}\xi_\nu A_0(u)^{-1}A_\nu(u) \tag{0.2}$$

has m distinct real eigenvalues $p_1(u, \xi), \cdots, p_m(u, \xi)$. We assume some of these eigenvalues actually depend on u because of quasi-linearity of the system (0.1).

We are interested in how hyperbolicity and non-linearity interact. To begin with, we seek an analogy of the oscillatory initial value problem which is basic in linear hyperbolic equations.

We choose as the initial data an m-vector of the form

$$u=\lambda^{-1}g(\lambda x\cdot\eta, x) \quad \text{at} \quad t=0, \tag{0.3}$$

where $\lambda>0$ is a large parameter, $x\cdot\eta$ the scalar product of x and $\eta\in\boldsymbol{R}^n$, η being a fixed n-vector $\neq 0$, and $g(s, x)$ is a given m-vector valued smooth function with compact support in s, x, i.e., $g\in C_0^\infty(\boldsymbol{R}^{n+1})^m$.

The following is a convenient assumption on the initial data:

$$\int_{\boldsymbol{R}}g(s, x)ds=0. \tag{0.4}$$

We may rewrite (0.3) as

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