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Moduli spaces of holomorphic mappings into hyperbolically imbedded complex spaces and hyperbolic fibre spaces

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1. Introduction.

Let Y be a complete hyperbolic complex space. We assume that Y is hyperbolically imbedded into an irreducible compact complex space $\bar{Y}$ as its Zariski open subset. Let X be a Zariski open subset of an irreducible compact complex space. We denote by $Hol(X, Y)$ (resp. $Mer_{dom}(X, Y)$) the set of all holomorphic (resp. dominant meromorphic) mappings of X into Y, where a mapping is said to be dominant if its image contains a nonempty open subset. In this paper, by making use of the theory developed by Noguchi [**12**, **13**, **16**] we study the structure of $Hol(X, Y)$. We first prove the following finiteness theorem for mappings in noncompact case, which was conjectured by Noguchi (cf. [**16**], Conjecture (5.5)),

FINITENESS THEOREM (cf. Theorem 2.3). *Let X and Y be as above. Then $Mer_{dom}(X, Y)$ is a finite set.*

This is regarded as the splitting case of the finiteness theorem of the sections of hyperbolic fibre spaces, and plays an essential role in considering the structure of hyperbolic fibre spaces in more general setting below. In the case of a noncompact quotient D/Γ of a bounded symmetric domain D in the complex vector space by a torsion free arithmetic discrete subgroup Γ of the identity component of the holomorphic automorphism group of D, D/Γ is complete hyperbolic and hyperbolically imbedded into its Satake compactification (cf. [**6**]). Thus applying the theorem to this case, we see the finiteness of $Mer_{dom}(X, D/\Gamma)$. Tsushima [**20**] obtained this result by showing the finiteness of dominant strictly rational maps into a smooth algebraic variety of log-general type. In the case where Y is a Riemann surface of finite type (g, n) with $2g-2+n>0$, Imayoshi [**2**] proved the above finiteness theorem. The compact version of the above theorem (a Lang's conjecture in [8]) was recently solved by Noguchi [**16**] (see § 2 for precise statement). Using this, Noguchi [**12**, **11**, **16**]