

Generalized standard Auslander-Reiten components

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Let A be an artin algebra, $\text{mod } A$ the category of finitely generated right A -modules, $\text{rad}^\infty(\text{mod } A)$ the infinite radical of $\text{mod } A$, and Γ_A the Auslander-Reiten quiver of A . It is well-known (see [6]) that Γ_A describes the quotient category $\text{mod } A/\text{rad}^\infty(\text{mod } A)$. We are interested in the behaviour of the connected components of Γ_A in the category $\text{mod } A$.

In the representation theory of finite dimensional algebras over an algebraically closed field k , an important role is played by the standard Auslander-Reiten components. Recall that following [12], [36], a connected component \mathcal{C} of the Auslander-Reiten quiver Γ_A of a finite dimensional k -algebra A is called standard if the full subcategory of $\text{mod } A$ formed by all modules from \mathcal{C} is equivalent to the mesh-category $k(\mathcal{C})$ of \mathcal{C} . If A is representation-finite (basic, connected), then Γ_A is *standard* if and only if A admits a simply connected Galois covering [12], [13]. Moreover, if k is of characteristic 2, then there are representation-finite, basic, connected k -algebras A with Γ_A nonstandard [32]. Examples of infinite standard components are the preprojective components, preinjective components and connecting components over representation-infinite tilted algebras as well as all tubes over tame tilted and tubular algebras [36], [24]. In the study of simply connected k -algebras of polynomial growth (in the sense of [39]) appeared a natural generalization of the notion of tube, called a coil, and then a more general concept of a multicoil, being a glueing of a finite number of coils by directed parts (see [3], [4], [41]). It is shown in [41] that a strongly simply connected k -algebra A is of polynomial growth if and only if every connected component of Γ_A containing an oriented cycle is a standard multicoil.

The aim of this paper is to introduce a natural generalization of the notion of standard component, called generalized standard component, which is simpler and makes sense for any artin algebra. We shall prove some basic facts on generalized standard Auslander-Reiten components and on artin algebras whose Auslander-Reiten quiver has such components. In particular, we solve (Corollary 2.5) a Ringel's problem [37, Problem 3] on the shape of regular standard