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Quasi-umbilical, locally strongly convex homogeneous affine hypersurfaces

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0. Introduction.

In this paper, we continue our investigations on homogeneous, locally strongly convex affine hypersurfaces in $\boldsymbol{R}^{n+1}$, which we started in [**DV1**] and [**DV2**].

A nondegenerate hypersurface M of the equiaffine space $\boldsymbol{R}^{n+1}$ is called locally homogeneous if for all points p and q of M, there exists a neighborhood U_p of p in M, and an equiaffine transformation A of $\boldsymbol{R}^{n+1}$, i.e. $A \in SL(n+1, \boldsymbol{R}) \ltimes \boldsymbol{R}^{n+1}$, such that $A(p)=q$ and $A(U_p) \subset M$. If $U_p=M$ for all p, then M is called homogeneous.

We denote the affine normal by ξ and the induced affine connection by ∇. We will always assume here that M is locally strongly convex. Let S denote the shape operator of the affine immersion. Since M is locally strongly convex, S is diagonalizable. If S is a multiple of the identity, M is called an affine sphere. Locally strongly convex homogeneous affine spheres have been studied in [**S**], see also the discussions in [**DV2**]. If the affine shape operator at each point has an eigenvalue λ with multiplicity exactly $n-1$, where n is the dimension of M, we call M proper quasi-umbilical. If $\lambda=0$ (so rank $(S)=1$), we recall the following result from [**DV1**].

THEOREM A [**DV1**]. *Let M be a locally strongly convex, locally homogeneous affine hypersurface with* rank $(S)=1$ *in $\boldsymbol{R}^{n+1}$. Then M is affine equivalent to the convex part of the hypersurface with equation*

$$\left(Z-\frac{1}{2}\sum_{i=1}^{r} X_i^2\right)^{r+2}\left(W-\frac{1}{2}\sum_{j=1}^{s} Y_j^2\right)^{s+2}=1,$$

where $r+s=n-1$ and $(X_1, \cdots, X_r, Y_1, \cdots, Y_s, Z, W)$ are the coordinates of $\boldsymbol{R}^{n+1}$.

Here, we will mainly consider the case that $\lambda \neq 0$. In Section 2, we will start to construct a special local tangent frame on a locally strongly convex,

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