Existence of positive entire solutions for higher order quasilinear elliptic equations

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1. Introduction.

This paper is concerned with the problem of existence of positive entire solutions for the $2M$-th order quasilinear elliptic equation

(1.1) $$(-\Delta)^M u = f(x, u, -\Delta u, \cdots, (-\Delta)^{M-1}u), \qquad x \in \mathbf{R}^N,$$

where $M \geq 2$, $N \geq 2$, Δ is the N-dimensional Laplace operator and $f \in C^\theta_{\text{loc}}(\mathbf{R}^N \times \mathbf{R}^M)$, $0 < \theta < 1$. An entire solution of (1.1) is defined to be a function u which is of class $C^{2M}(\mathbf{R}^N)$ and satisfies (1.1) at every point of $\mathbf{R}^N$.

Beginning with Kusano and Swanson [10], several authors have developed existence theory of *radial* entire solutions for higher order elliptic equations of the type (1.1) with radial symmetry; see e. g. the papers [1, 2, 6-10]. A natural question then arises: Is it possible to construct *non-radial* entire solutions of the equation (1.1) without radial symmetry? An answer to this question has been given by Edelson and Vakilian [3] and Kusano and Swanson [11], who have examined the equation $(-\Delta)^M u = f(x, u)$ by employing entirely different methods. A principal tool used in the paper [11] is an extension of the supersolution-subsolution method (super-subsolution method for short) which has proved to be very powerful in establishing the existence of entire solutions for second order elliptic equations of the form $-\Delta u = f(x, u)$. Such an extension in [11] relies on the derivation of a super-subsolution principle holding for second order elliptic systems of the form

(1.2) $$-\Delta u_i = f_i(x, u_1, \cdots, u_M), \qquad x \in \mathbf{R}^N, \ i = 1, 2, \cdots, M.$$

It will be natural to expect that the super-subsolution principle for (1.2) given in [11] could be generalized so as to give rise to a new super-subsolution method for constructing non-radial entire solution of (1.1), thereby generalizing considerably the results of [3] and [11]. The purpose of this paper is to verify