J. Math. Soc. Japan
Vol. 46, No. 3, 1994

Formal neighbourhoods of a toric variety and unirationality of algebraic varieties

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(Received July 6, 1992)

§0. Introduction.

This paper provides a partial answer to the following question.

QUESTION. *Assume that a nonsingular complete algebraic variety X of dimension $n+2$ $(n\geqq 1)$ contains a nonsingular rational surface S with $N_{S/X}$ ample. Then is X unirational?*

In the previous paper [2], we have solved this question affirmatively in the case where $n=1$ (i.e., X is three-dimensional) and S is toric. Here we shall generalize the main theorem of [2] to the higher-dimensional case. The main result is the following.

MAIN THEOREM. *Let n be a positive integer. Let X be a nonsingular complete algebraic variety of dimension $n+2$. Assume that X contains a nonsingular projective toric surface S and that the following two conditions* (a) *and* (b) *are satisfied*:

(a) $N_{S/X}\cong\bigoplus_{\mu=1}^{n} A_{\mu}$ *where each A_{μ} is an ample line bundle,*

(b) $H^1(S, N_{S/X}\otimes S^q(N_{S/X}^{\vee}))=0$ *for each positive integer q.*

Then X is unirational.

As is easily seen, this theorem is a generalization of the main theorem of [2]. The following corollary would be a help to understanding of the statement of Main Theorem.

COROLLARY. *Let X be a nonsingular complete algebraic variety of dimension $n+2$ and L a line bundle on X. Assume that there exists a sequence*

$$X=X_0\supset X_1\supset\cdots\supset X_n=S$$

of subvarieties of X satisfying the following three conditions:

(1) *X_i is a smooth member of the linear system $|L|_{X_{i-1}}|$ on X_{i-1} $(1\leqq i\leqq n)$,*

(2) *$X_n=S$ is a toric surface,*

This work is partially supported by the Fûjukai Foundation.