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Recurrence conditions for multidimensional processes of Ornstein-Uhlenbeck type

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1. Introduction and results.

A stochastic process of Ornstein-Uhlenbeck type on the Euclidean space is a Markov process obtained from a spatially homogeneous Markov process undergoing a linear drift force determined by a matrix $-Q$. We give a criterion of recurrence and transience for a process of this type under the assumption that Q is diagonalizable and its eigenvalues are positive. No restriction is imposed on the part of the spatially homogeneous Markov process.

Rigorous definition of our process is as follows. Let $\boldsymbol{G}$ be an operator defined by

$$\begin{aligned}(1.1)\qquad \boldsymbol{G}f(x)= &\sum_{j=1}^{d} a_j D_j f(x)+\frac{1}{2}\sum_{j,k=1}^{d} B_{jk}D_jD_kf(x)\\ &+\int_{\boldsymbol{R}^d}\left[f(x+y)-f(x)-\sum_{j=1}^{d}\frac{y_j}{1+|y|^2}D_jf(x)\right]\rho(dy)\\ &-\sum_{j,k=1}^{d}Q_{jk}x_kD_jf(x),\end{aligned}$$

where D_j stands for partial derivative in x_j. Here $a=(a_j)$ is a constant vector, $B=(B_{jk})$ is a symmetric nonnegative-definite constant matrix, ρ is a measure on $\boldsymbol{R}^d$ with $\rho(\{0\})=0$ and $\int|y|^2(1+|y|^2)^{-1}\rho(dy)<\infty$, and $Q=(Q_{jk})$ is a constant matrix. We consider the real Banach space $C_0(\boldsymbol{R}^d)$ of continuous functions vanishing at infinity with the norm of uniform convergence. The operator $\boldsymbol{G}$ is acting in this space and its domain is the class of C^2 functions with compact supports. It is proved in Sato and Yamazato [**10**] that the smallest closed extension $\bar{\boldsymbol{G}}$ of $\boldsymbol{G}$ is the infinitesimal generator of a strongly continuous nonnegative contraction semigroup on $C_0(\boldsymbol{R}^d)$. So a Markov process $\boldsymbol{X}$ on $\boldsymbol{R}^d$ is associated and it is represented, as usual (see [**1**]), by $(\Omega, \mathcal{F}, \mathcal{F}_t, \boldsymbol{P}^x, X_t)$ with $\boldsymbol{P}^x(X_0=x)=1$. The Markov process $\boldsymbol{X}$ is called in [**10**] the process of Ornstein-Uhlenbeck type associated with $\boldsymbol{G}$. The measure ρ is called the Lévy measure