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Notes on the mean value property for certain degenerate elliptic operators

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Introduction.

The purpose of this paper is to study the mean value property and its applications to a certain class of degenerate elliptic operators. We shall treat the operators L_{α} defined by

$$L_{\alpha}(x, \partial_{x}) = -x_{n}\Delta - \alpha\partial_{x_{n}} \quad \text{for} \quad x = (x', x_{n}) \in \boldsymbol{R}_{+}^{n}, \tag{0.1}$$

where α is a real parameter and $\boldsymbol{R}_{+}^{n}$ is the Euclidian halfspace defined by $\{x=(x', x_{n}) | x' \in \boldsymbol{R}^{n-1}, x_{n}>0\}$.

Let Ω be a domain of $\boldsymbol{R}_{+}^{n}$ and we set

$$\begin{aligned} \underline{\Omega} &= \Omega \cup (\partial\Omega \cap \partial\boldsymbol{R}_{+}^{n}), \\ \partial\Omega^{i} &= \partial\Omega \setminus \partial\boldsymbol{R}_{+}^{n}. \end{aligned} \tag{0.2}$$

By $C^{0}(\Omega)$ and $C^{0}(\underline{\Omega})$ we denote the sets of all continuous functions on Ω and $\underline{\Omega}$ respectively.

With the operators L_{α}, we shall associate the modified mean value functions $M_{\alpha,\rho}u(a)$ of $u \in C^{0}(\Omega)$ (resp. $u \in C^{0}(\underline{\Omega})$) at a point $a \in \Omega$ (resp. $a \in \underline{\Omega}$). More precisely

DEFINITION 0.1. Let $a=(a', a_{n})$ be an arbitrary point in Ω (resp. $\underline{\Omega}$), and let α and ρ be arbitrary positive numbers satisfying $\rho<\operatorname{dist}(a, \partial\Omega)$ (resp. $\rho<\operatorname{dist}(a, \partial\Omega^{i})$). For $u \in C^{0}(\Omega)$ (resp. $u \in C^{0}(\underline{\Omega})$) we set

$$\begin{aligned} &M_{\alpha,\rho}u(a) \\ &\quad = C(\alpha)\rho^{1-n-\alpha}\int_{0}^{1}\{s(1-s)\}^{\alpha/2-1}ds\int_{\partial B_{\rho}^{+}} x_{n}^{\alpha}u(x'+a', \gamma(x_{n}, a_{n}, s))dS_{x} \end{aligned} \tag{0.3}$$

where

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