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Nonradial solutions of semilinear elliptic equations on annuli

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1. Introduction.

Let $\Omega_a=\{x\in\boldsymbol{R}^N : a<|x|<a+1\}$ with $a>0$ and $g:\boldsymbol{R}\to\boldsymbol{R}$ is continuous. We are concerned with the problem

$$\begin{cases} -\Delta u = \lambda u+g(u) & \text{in } \Omega_a \\ u=0 & \text{on } \partial\Omega_a \\ u>0 & \text{in } \Omega_a . \end{cases} \tag{1}$$

It was known that any solution of (1) is radially symmetric in the case that the domain Ω_a is a disk instead of an annulus in Gidas, Ni and Nirenberg [**4**].

On the other hand, the existence of nonradial solutions of (1) in an annulus Ω_a was first obtained when $\lambda=0$, $g(t)=t^p$ with p close to $(N+2)/(N-2)$ and $N\geqq 3$ by Brezis and Nirenberg [**2**]. Later Coffman [**3**] showed the generation of essentially infinitely many nonradial solutions as $a\to+\infty$ for $\lambda=-1$ and $g(t)=t^p$, where $N=2$ and $1<p<\infty$. This result was generalized by Kawohl [**7**] and Suzuki [**11**] in the case of $N=2$ and then by Li [**9**] when $N\geqq 4$. These arguments can be applied only to the case of homogeneous nonlinearities or nonlinear eigenvalue problems because the Lagrangean multiplier principle played a crucial role there.

In order to prove the existence of nonradial solutions of the problem (1) with a general nonlinearity, Lin [**10**] used a spectral analysis for solutions produced by the Nehari variation and Suzuki [**12**] was based on estimates the critical values obtained by the mountain pass lemma.

Our purpose of the present paper is to give a simple proof of the above results. We make use of estimates of the Morse indices of the critical points given by the mountain pass lemma, which was first employed to get a sequence of subharmonic solutions of an elliptic equation on a strip-like domain in [**5**]. Our method enables us to weaken the growth condition of the nonlinearity g of (1) because we do not need information about the order of critical values as $a\to+\infty$.