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An inverse problem in bifurcation theory, II

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1. The main result.

We consider the nonlinear boundary value problem

$$(1.1)\qquad \begin{cases} \dfrac{d^2u}{dx^2}+\lambda u=uf(u), & 0<x<\pi, \\ u(0)=u(\pi)=0, \end{cases}$$

where λ is a real parameter and f is a Lipschitz continuous, real function defined on $\boldsymbol{R}$. Without loss of generality, we assume that $f(0)=0$. By a solution of (1.1) we mean a pair $(\lambda, u)\in\boldsymbol{R}\times C^2[0, \pi]$ satisfying (1.1). Let $\Gamma_n(f)$, $n=1, 2, \cdots$, denote the set of $(\lambda, h)\in\boldsymbol{R}^2$ for which there exists a solution (λ, u) of (1.1) satisfying the following conditions:

(i) $u(x)$ has exactly $n-1$ zeros in $(0, \pi)$;

(ii) The first stationary value of $u(x)$ is equal to h.

The set $\Gamma_n(f)$ is considered to be a representation in $\boldsymbol{R}^2$ of a set of nontrivial solutions of (1.1) bifurcating from the trivial solution $(n^2, 0)$ (note that n^2 is the n-th eigenvalue of the linearized problem of (1.1)).

In the previous paper [2] the author established a result that a nonlinear term f is determined uniquely from its solution set $\Gamma_1(f)$ and, in particular, that

$$\Gamma_1(f)=\{(1, h)\in\boldsymbol{R}^2 : h\neq 0\}$$

implies $f\equiv 0$. The purpose of the present paper is to show that a nonlinear term f is not determined uniquely by the condition

$$(1.2)\qquad \Gamma_2(f)=\{(4, h)\in\boldsymbol{R}^2 : h\neq 0\}$$

and to find nonlinear terms f satisfying the condition (1.2).

To state our result precisely we need some terminology. Let $0\leqq\alpha\leqq 1/2$ and let X_+ be the function space

$$(1.3)\qquad X_+ := \{g(h)\in C^1[0, \infty) : g(0)=0;\ |g|_0+|g'(0)|+|g'|_\alpha =: \|g\|_{X_+}<\infty\},$$

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