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Ideal points and incompressible surfaces in two-bridge knot complements

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§ 0. Introduction.

An ideal point will be a limit of representations of a fundamental group Γ of a three manifold in PSL_2C. In [6] Culler and Shalen constructed incompressible surfaces of a three manifold from ideal points via Serre's tree. However, it is hard to understand Serre's tree from topological viewpoint because its definition is purely algebraic.

In this paper we construct Serre's trees concretely for two-bridge knot complements. We regard Serre's tree as the way how the geodesics in $\boldsymbol{H}^3$ fixed by γ ($\in\Gamma$) converge, as in Lemma 3.1. With this observation we can guess a rough shape of Serre's tree by describing geodesic in $\boldsymbol{H}^3$ with computer graphics for representations near an ideal point. This observation is also useful in determining shapes of trees with fine prospects. However, to give a precise proof we cannot use Lemma 3.1 and we need to construct trees step by step using Γ-tree's arguments. As the results, we classify the ideal points for two-bridge knot complements, and determine the complete correspondence between the ideal points and incompressible surfaces which are classified in [7].

In § 1 we show that PSL_2C representation space of a two-bridge knot group is a punctured Riemann surface, as is studied by Riley in [9]. We use here the method of Burde [2], in which $SO(3)$ representation spaces are discussed. Section 2 recalls the definition of Serre's tree and the classification of incompressible surfaces of a two-bridge knot complement. In sections 3 and 4 we examine Serre's trees for the ideal points of a two-bridge knot complement. In § 5 we summarize the main results. We apply the results to obtain a new proof of the fact that two-bridge knots have property P, which was proved by Takahashi [12] in 1981 and by Burde [2] in 1987. In § 6 we give proofs of lemmas used in § 4.

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