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Structure theorems for positive radial solutions to $\operatorname{div}(|Du|^{m-2}Du)+K(|x|)u^q=0$ in $\boldsymbol{R}^n$

Dedicated to Professor Takaŝi Kusano on his 60th birthday

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§1. Introduction.

In this paper we investigate the structure of positive radial solutions to the following quasilinear elliptic equation

$$\text{(E)} \qquad \operatorname{div}(|Du|^{m-2}Du)+K(|x|)u^q=0\,, \qquad x\in\boldsymbol{R}^n\,,$$

where $q>m-1$, $n>m>1$, and $|x|=\{\sum_{i=1}^n x_i^2\}^{1/2}$. When $m=2$, the equation (E) reduces to the semilinear elliptic equation

$$\Delta u+K(|x|)u^q=0\,.$$

Recently, in Theorem 1 of [**KYY**], we have established a structure theorem for positive radial solutions of the latter equation. (See also [**Y1**] and [**Y2**].) The aim of this paper is to show that the result is extended to the equation (E).

Since we are only concerned with positive radial solutions (i.e., solutions with $u(x)=u(|x|)>0$ for all $x\in\boldsymbol{R}^n$), we will study the initial value problem

$$(\mathrm{K}_\alpha) \qquad \begin{cases} (r^{n-1}|u_r|^{m-2}u_r)_r+r^{n-1}K(r)(u^+)^q=0\,, & r>0\,, \\ u(0)=\alpha>0\,, \end{cases}$$

where $r=|x|$ and $u^+=\max\{u, 0\}$. We assume that

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