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Almost coinciding families and gaps in $\mathcal{P}(\omega)$

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1. Introduction.

In [7], S. Mardesic and A. Prasolov translate the calculation of the k-dimensional strong homology of $Y^{(k+1)}$ (the discrete sum of countably many copies of the $(k+1)$-dimensional Hawaiian earring) into a condition which is the existence of a certain family of functions on ω. They showed that the k-dimensional strong homology of $Y^{(k+1)}$ is nontrivial if and only if there exists such a nontrivial family. After that, Dow, Simon and Vanghan [4] named these families almost coinciding families and showed the following Proposition 1~3.

PROPOSITION 1. [4, Theorem 2.4] *If $\mathbf{d}=\omega_1$, then there exists a nontrivial almost coinciding family indexed by ${}^{\omega}\omega$.*

PROPOSITION 2. [4, Theorem 3.1] *The proper forcing axiom implies that every almost coinciding family indexed by ${}^{\omega}\omega$ is trivial.*

PROPOSITION 3. [4, Theorem 4.1, Lemma 4.2, 4.3] *If there exists a nontrivial almost coinciding family indexed by ${}^{\omega}\omega$, then there exists an unfilled $(\mathbf{b}, \mathbf{b})$-gap in $\mathcal{P}(\omega)$. So, in Kunen's model of "*$\mathbf{ZFC}$+*Martin's Axiom* ($\mathbf{MA}$)$+2^{\omega}=\omega_2$+*there are no unfilled $(2^{\omega}, 2^{\omega})$-gaps", there does not exist a nontrivial almost coinciding family indexed by ${}^{\omega}\omega$.*

By Propositions 1 and 2, the existence of nontrivial almost coinciding families is independent from the negation of the Continuum Hypothesis (**CH**). It is an interesting problem to consider whether certain set theoretical axioms imply the existence of nontrivial almost coinciding families. In this paper, we shall show

THEOREM 1. *Let P be the partially ordered set (poset for short) which adjoins ω_2 Cohen reals. Then, in V^P, these does not exist a nontrivial almost coinciding family indexed by ${}^{\omega}\omega$.*

THEOREM 2. *Let $\omega_1<\kappa=\kappa^{<\kappa}$. Then, there is a poset P with the countable chain condition such that, in V^P, $2^{\omega}=\kappa+$*$\mathbf{MA}$*+there exists an unfilled (κ, κ)-gap+*