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On Nikulin's theorem on fixed components of linear systems on K3 surfaces

Dedicated to Professor Heisuke Hironaka on his sixtieth birthday

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§ 0. Introduction.

In this article we would like to give more natural verification to Nikulin's theorems in Nikulin [4] and Nikulin [5]. (Nikulin [4] is a report distributed at the symposium held at Tokyo Metropolitan University in August 1990.)

First we explain Nikulin's results.

Let X be a K3 surface (i.e., a smooth projective algebraic surface over algebraically closed field such that the canonical line bundle K_X is trivial and $H^1(X, \mathcal{O}_X)=0$). A line bundle L (resp. a divisor D) on X is *nef*, if $L\cdot C\geqq 0$ (resp. $D\cdot C\geqq 0$) holds for every irreducible algebraic curve C on X. Needless to say, the self-intersection number $C\cdot C=C^2$ is an even integer with $\geqq -2$ for every irreducible curve C. $C^2=-2$ if and only if C is a smooth rational curve. Moreover, $h^0(\mathcal{O}_X(C))=\dim H^0(\mathcal{O}_X(C))=\dim|C|+1=C^2/2+2$ by Riemann-Roch theorem.

Here we quote the following proposition contained in [4] and [5]. It follows easily from Saint-Donat [6].

PROPOSITION 0.1. *Let* $H\in \mathrm{Pic}(X)$ *be a nef line bundle. One of the following cases* (1)-(4) *holds.*

(1) $H^2>0$. *The complete linear system* $|H|$ *contains an irreducible curve and has no fixed point.* $\dim|H|=H^2/2+1>0$.

(2) $H^2=0$, $|H|=m|E|$, $m>0$, *where* $|E|$ *is an elliptic pencil.* ($|H|$ *contains an irreducible curve for* $m=1$ *only.*)

(3) $H^2>0$, $|H|=m|E|+\Gamma$, $m\geqq 2$, *where* $|E|$ *is an elliptic pencil and* Γ *is an irreducible curve with* $\Gamma^2=-2$ *and* $E\cdot\Gamma=1$. *Here also* $m=\dim|H|=H^2/2+1$. Γ *is the fixed part of* $|H|$.

(4) $H\cong\mathcal{O}_X$, $|H|=\{\emptyset\}$.

Let $\Delta=\sum n_i\Delta_i$ be an effective divisor on X. By $G(\Delta)$ we denote the dual graph of intersections of the components Δ_i of Δ, the weight of the vertex corresponding to Δ_i being the multiplicity n_i, the number of edges connecting