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On the decomposition of conformally flat manifolds

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1. Introduction.

Let M be a smooth n-manifold and C a conformal class on M. (M, C) is *conformally flat* if for any point p of M, there exists a metric g contained in C such that g is flat on some neighborhood of p. A conformal class C is called a *flat conformal structure* if (M, C) is conformally flat. A manifold M is said to be *conformally flat* if M admits a flat conformal structure. In this paper, we always assume a manifold M to be smooth, compact and connected with $\dim M=n\geqq 3$, unless otherwise stated. For an orientable manifold M, we also assume that M is oriented.

DEFINITION 1.1. An n-manifold M is said to be *nontrivial* if M is not diffeomorphic to the standard n-sphere S^n. And M is *C-prime* if

(1) M is non-trivial and conformally flat, and

(2) there is no decomposition $M=M_1\# M_2$ (a connected sum of M_1 and M_2), where each of M_1 and M_2 has the property (1).

A well-known theorem of Kulkarni [12] states that a connected sum of conformally flat manifolds is also conformally flat. Thus, connected sums of C-prime manifolds are conformally flat. On the other hand, a simple observation gives the following proposition.

PROPOSITION 2.1. *Every non-trivial conformally flat manifold is diffeomorphic to a connected sum of a finite number of C-prime manifolds.*

Thus the classification problem of conformally flat manifolds is reduced to the classification of C-prime manifolds. A decomposition $M=P_1\#\cdots\# P_k$, where each P_i is C-prime, is called a *C-prime decomposition* of M in this paper.

The purpose of this paper is to show several results concerning the C-prime decomposition of conformally flat manifolds. In section 2 we prove Proposition 2.1 above and some sufficient conditions for a manifold to be C-prime. We also discuss the Yamabe invariant $\mu(M, C)$ (see Definition 2.4) of a conformally flat manifold (M, C). And we see that, for some M, there exists a sequence of flat conformal structures on M, which maximizes the Yamabe invariant, such that the limit of this sequence gives a decomposition of M.