J. Math. Soc. Japan
Vol. 45, No. 1, 1993

Certain polynomials for knots with integral representations

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(Received Nov. 26, 1991)
(Revised Dec. 25, 1991)

In this paper we define a polynomial with integer coefficients for knots with certain integral representations of the knot group. This polynomial is determined up to power by the conjugacy class of the representation and is related to Dehn surgery on knots as follows: if $(1/m)$-surgery on such a knot yields a homotopy 3-sphere, then the absolute value of that polynomial at m must be equal to 1. The degree of the polynomial is related to the class number of the algebraic number field of the representation. We thus have a rough estimate on the number of Dehn surgeries of a given knot yielding simply-connected manifolds from the class number of the algebraic number field. As an example, we will calculate this polynomial for some knots and show that Property P actually follows from the polynomial for those knots, i.e. the polynomials do not take the value ± 1 at non-zero integers for those knots.

The knots for which we will define the polynomial form a large class, and we will call them integral knots. In fact, if a nontrivial knot has no essential closed surfaces in the complement, then that knot is an integral knot. The definition of integral knots we shall use in this paper is the following.

DEFINITION. Let J be a smooth knot in the 3-sphere S^3. We say J is an integral knot if there is an algebraic number field k, i.e., a finite extension of the rationals $\boldsymbol{Q}$ in $\boldsymbol{C}$, and a representation

$$\rho : \pi_1(S^3 - J) \longrightarrow PSL_2(\mathcal{O}_k),$$

where $\mathcal{O}_k$ is the ring of algebraic integers in k, such that:

(1) ρ is a parabolic representation in the sence that if $\gamma \in \pi_1(S^3 - J)$ is a peripheral element, then $\rho(\gamma) = \pm(\textit{unipotent matrix})$, and

(2) ρ is irreducible over the complex numbers, i.e., ρ is not conjugate in $PSL_2(\boldsymbol{C})$ to a group of upper-triangular matrices.

We will call (J, ρ) a k-integral knot to specify the algebraic number field k.

Integral knots form a large class. In fact we have the following fact which

This research was partially supported by Grant-in-Aid for Scientific Research (No. 04740012), Ministry of Education, Science and Culture.