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Unirational elliptic surfaces in characteristic 2

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§ 0. Introduction.

Let X be a non-singular projective surface defined over an algebraically closed field k of characteristic p. X is called unirational if there exists a generically surjective rational mapping from the projective space $\boldsymbol{P}^2$ to X. In characteristic 0, the unirationality of an algebraic surface is equivalent to the rationality. In general, however, there exist irrational unirational algebraic surfaces, so, it is interesting to characterize unirational surfaces over an algebraically closed field of characteristic $p>0$, a problem which many people have been concerned with. From this point of view, T. Katsura [**4**] has completely determined irrational elliptic surfaces with sections which are unirational and of base change type (for definition, see Definition 1.1) in the case where the characteristic p is more than two. The objective of this paper is to give a similar result in characteristic two.

THEOREM 0.1. *Let k be an algebraically closed field of characteristic 2, and $\boldsymbol{P}^1$ the projective line with the function field $k(\boldsymbol{P}^1)=k(t)$. Any minimal Weierstrass normal form (for definition, see e.g. p. 171 [**17**]) of an irrational unirational elliptic surface $f: X \to \boldsymbol{P}^1$ of base change type with sections over k is given by one of the following:*

$(b_j,\ c_j,\ d_j \in k)$

(1) $$y^2+t^6y = x^3+t(b_0t^3+b_1t^2+b_2t+b_3)t^3+c_0t^6x+d_0t^{11}, \quad (b_3,\ c_0 \neq 0),$$

(2) $$y^2+t^6y = x^3+t(b_0t^3+b_1t^2+b_2t+b_3)x^2, \quad (b_3 \neq 0),$$

(3) $$y^2+t^2y = x^3+t(b_0t+b_1)x^2+t^3(c_0t^4+c_1t^2+c_2), \quad (c_0,\ c_2 \neq 0),$$

(4) $$y^2+t^2y = x^3+t(b_0t^2+b_1t+b_2)x^2+c_0t^4x+t^3(d_0t^2+d_1), \quad (b_0,\ c_0,\ d_1 \neq 0),$$

(5) $$y^2+t^2y = x^3+t(b_0t^2+b_1t+b_2)x^2+c_0t^2x+d_0t^3, \quad (b_0,\ b_2c_0+d_0 \neq 0),$$

(6) $$y^2+t^2y = x^3+b_0t^2x^2+t^5(c_0t+c_1), \quad (c_0 \neq 0),$$

(7) $$y^2+t^2y = x^3+t^2(b_0t+b_1)x^2+c_0t^4x+d_0t^5, \quad (b_0,\ c_0 \neq 0),$$