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Fourth order quasilinear evolution equations of hyperbolic type

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1. Introduction.

Some vibratory phenomena of beams may be described by the fourth order quasilinear evolution equation

$$(\partial_t^2+A_2(t,\ u))\cdot(\partial_t^2+A_1(t,\ u))u+G(t,\ u,\ \partial_t u,\ \partial_t^2 u,\ \partial_t^3 u)=0 \qquad (t>0), \tag{1.1}$$

where $A_i(t, u)$, $i=1, 2$, are (unbounded) self-adjoint positive definite operators in a Hilbert space H, and G is a lower order nonlinear perturbation.

In such a generality, (1.1) is not so easy to be dealt with. One could imagine that it is possible to reduce it to a first order equation in a 4-ple of Hilbert space, and then apply known theories (see e.g. [**K**]). However in this case those methods seem to be too hard to be handled.

In [**P**], [**AP**] a very special semilinear case was studied by an *ad hoc* method, which provided global existence and boundedness of the solutions of the Cauchy problem.

As a preparation for the study of (1.1) here we confine ourselves to study the local well-posedness of the Cauchy problem for the equation

$$(\partial_t^2+\gamma_2(u)A)\cdot(\partial_t^2+\gamma_1(u)A)u=0 \qquad (t>0), \tag{1.2}$$

where A is an (unbounded) self-adjoint positive definite operator in H, and γ_i ($i=1, 2$) are real functionals on $D(A)$, the domain of A.

THEOREM 1.1 (Main result). *Let A be an (unbounded) self-adjoint positive definite operator in a Hilbert space H, with inner product $(\cdot, \cdot)$.*

For $i=1, 2$ let m_i: $[0, +\infty[\rightarrow[0, +\infty[$ satisfy:

i) *m_i is thrice continuously differentiable* ($i=1, 2$);

ii) $m_i(r)\geqq\nu>0 \qquad (r\geqq 0;\ i=1, 2)$;

iii) $|m_2(r)-m_1(r)|\geqq\delta>0 \qquad (r\geqq 0)$.

Then the Cauchy problem for the equation