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Localization of diffusion processes in one-dimensional random environment[1)]

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Introduction.

Let $X(t, W)$ be a one-dimensional diffusion process starting at 0 and with generator

$$\mathcal{L}_W = \frac{1}{2} e^{W(x)} \frac{d}{dx}\left(e^{-W(x)} \frac{d}{dx}\right), \tag{1}$$

where $\{W(x), x \in \boldsymbol{R}\}$ is a random environment. The process $X(t, W)$ can be constructed from a one-dimensional Brownian motion through a change of scale and time. It is assumed that the Brownian motion (used for the construction of $X(t, W)$) and the random environment $\{W(x)\}$ are independent. *Formally*, $X(t, W)$ is a solution of the stochastic differential equation

$$dX(t) = \text{Brownian differential} - \frac{1}{2} W'(X(t))dt.$$

We are interested in the asymptotic properties of $X(t, W)$ as $t \to \infty$. A result for this type of random environment problem goes back to Sinai [**12**]. When $\{W(x), x \in \boldsymbol{R}\}$ is a Brownian environment, Brox [**1**] introduced the diffusion process $X(t, W)$ as a continuous model of Sinai's random walk ([**12**]) in a Bernoulli environment and obtained the following result of Sinai-type: $(\log t)^{-2} X(t, \cdot) - b(t, \cdot)$ tends to 0 in probability as $t \to \infty$ where $b(t, W)$ is a suitable function depending only on t and the environment $W = W(\cdot)$; moreover, the distribution of $(\log t)^{-2} X(t, \cdot)$ tends to a limit which is the same as the limit distribution in Sinai's case. Kesten [**9**] and Golosov [**5**] obtained the explicit form of the limit distribution (see also [**15**] for some extension). Results of Sinai-type for a wider class of random environments were then obtained by Letchikov [**10**] (for non-simple random walks) and Kawazu, Tamura and Tanaka [7], [8] (for

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